Perspectives on Deepening Teachers' Mathematics and Science Content Knowledge

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Teacher content knowledge plays an integral role in the quality of mathematics and science education experienced by students. Teachers' content knowledge impacts not only what teachers teach, their knowledge also influences how they teach and what students learn (e.g., Gess-Newsome & Lederman, 1995; Sowder, Phillip, Armstrong, & Schappelle, 1998). The content knowledge of effective mathematics and science teachers is not limited to their knowledge of the discipline. Teacher content knowledge includes a complex set of understandings that guides their work with students (Ball & Bass, 2000; Magnusson, Krajcik, & Borko, 1999).

Designers of professional development for mathematics and science teachers are faced with many decisions related to the content they choose to address, the selection of strategies to reach professional development goals, and how they facilitate transfer of teachers' new knowledge to their classroom practice. Many of the Math Science Partnership (MSP) projects funded by the National Science Foundation (NSF) include professional development activities to deepen mathematics/science teacher knowledge. The MSP Knowledge Management and Dissemination project was funded by NSF to integrate what the MSP projects were learning into the larger knowledge base about improving mathematics and science education. As part of this work, we have created this volume of "stories" of professional development to illustrate some of the choices MSP projects have made and how their decisions played out over time. It is our hope that designers of future professional development endeavors will find these stories, and the authors' lessons learned, useful when considering their design options.

FACETS OF TEACHER CONTENT KNOWLEDGE*

A new era of scholarly and practical work on teacher content knowledge emerged in 1986 when Shulman proposed five content-specific domains of teacher knowledge: the content itself (facts and concepts of the discipline), the substantive structures of the discipline (how knowledge is organized), the syntactic structures of the discipline (ways of knowing in the discipline), pedagogical content knowledge, and curriculum content knowledge. Although Shulman described these teacher knowledge domains as content-specific, he proposed them without reference to any particular content area.

Over the past 20 years, scholars have suggested a number of categories and characterizations to further clarify teacher content knowledge in mathematics and science. Theoretical writings and empirical studies have given considerable attention to pedagogical content knowledge, in particular. More recently, the field's attention has turned to defining more specifically the disciplinary content knowledge that teachers need for effective practice. The result is a substantial body of work that describes a variety of content-specific knowledge types. To make sense of this diverse body of literature, we consider three primary facets:

- Disciplinary content knowledge;
- Ways of knowing in the discipline; and
- Pedagogical content knowledge.

* This text is drawn from the "Defining Teacher Content Knowledge" Knowledge Review published on the MSP KMD website, available online at http://www.mspkmd.net/blasts/tck.php

Most discussions of teacher content knowledge do not include beliefs as knowledge, but it is important to recognize that beliefs, such as whether and why certain topics are important or what the goals of instruction should be, mediate how teachers draw on their knowledge in their work. One teacher may see the primary aim of instruction as developing conceptual understanding, and draw on content knowledge to structure lessons that engage students with experiences that challenge their naive understandings. In contrast, a teacher who sees the purpose of instruction as transmitting the facts and algorithms of a discipline is likely to draw on content knowledge to structure lessons very differently. Rarely, if ever, is one category of teacher content knowledge proposed as the only one that matters. Few would argue that any one of them is unimportant for teaching. However, specific instances of research or practice often emphasize one or more of these facets, or a particular component within one of them. A brief description of each of the three primary facets of teacher content knowledge we are considering follows, as well as some distinctions among differing aspects within each of these facets.

Disciplinary Content Knowledge

Disciplinary content knowledge can be thought of as an individual's understanding of subject matter concepts and how these concepts relate to form the larger body of knowledge. Hill & Ball (2004) refer to this kind of knowledge as "common knowledge of content;" Ferrini-Mundy and colleagues (Ferrini-Mundy, Floden, McCrory, Burrill, & Sandow, 2005) call it "core content knowledge." Another way to think about common or core knowledge of disciplinary content is as the knowledge held by all professionals who use science or mathematics in their work. This kind of knowledge is not unique to teachers, but it definitely has important connections to teaching.

There is general agreement that teachers need to know the disciplinary content they teach, but there are multiple points of view on what it means to know that content, or what is the appropriate way for teachers to know it. These views tend to fall into three different levels of content knowledge: (1) knowledge of student-level disciplinary content; (2) knowledge of advanced disciplinary content; and (3) knowledge of profound disciplinary content.

Knowledge of Student-level Disciplinary Content

One level of disciplinary content knowledge is teachers' own understanding of the content they are expected to teach at a particular grade level. The nature and scope of content teachers are expected to teach throughout the K–12 curriculum has changed substantially over the last 15 years, especially evident in the "standards movement" at the national and state levels. Developers of instructional materials have responded to the changing content in national and state standards by adding new topics or moving topics from one grade level to another. The upshot is that teachers may be unfamiliar with content ideas they are required to teach at their own grade level. The argument for developing teachers' knowledge of disciplinary content at the student level emphasizes the importance of teachers understanding the content they are expected to teach, at least at the same depth students are expected to attain at that grade.

The teacher content knowledge literature frequently *acknowledges* the necessity of teachers having grade-level content knowledge (e.g., Ferrini-Mundy et al., 2005; Carlsen, 1999), but this level of knowledge has not typically been the *direct focus* of scholarly work because its necessity is generally taken for granted. In practice, given the limited time and resources that can be devoted to deepening teachers' content knowledge, many efforts may be unable to go much

further than addressing grade-level content. Additionally, key decision-makers within a professional development effort may differ in their views about teacher content knowledge, sometimes even without realizing it. As a result, they may negotiate to focus primary attention on the grade-level content knowledge that fits across their views. Even if it represents a kind of compromise, attention to student-level disciplinary content knowledge is evident in many situations and deserves separate treatment when considering the content knowledge teachers need for effective teaching.

Knowledge of Advanced Disciplinary Content

A common view is that teachers' mathematics and science content knowledge must extend well beyond the content that a teachers' students are expected to know. In this view, the content knowledge of key importance is understanding the fundamental ideas of the mathematics and science disciplines (Askey, 1999; Cuoco, 2001; Tracy & Walsh, 2004; Wu, 1997). This view of teacher content knowledge is closely associated with the position that K–12 education should establish the foundation for students to develop both knowledge and appreciation of the major concepts and unifying ideas in mathematics and science. The Conference Board of the Mathematical Sciences (2001) recommended that teacher preparation develop in teachers, "a thorough mastery of the mathematics in several grades beyond that which they expect to teach." Similarly, in its position statement on science teacher preparation, the National Science Teachers Association (NSTA, 2004) strongly recommended that programs enable prospective science teachers to, "Develop robust science knowledge and skills beyond the depth and breadth needed for teaching a curriculum based on the National Science Education Standards at the grade levels they are preparing to teach" (p. 1).

A missing element in the preparation of many teachers is the opportunity to study the mathematics and science content of the K–12 curriculum beyond their own experience in K–12 schools. Discussing the preparation of secondary mathematics teachers, Wu (1999, p. 8-9) called for courses "which do not stray far from the high school curriculum" in order to "revisit all the standard topics in high school from an advanced standpoint." This advanced standpoint includes attending to the historical background of major ideas, inter-connections among them, and complete and rigorous proofs in mathematics or lines of evidence in science. The essential argument is that teacher learning of content that is more advanced than what students are expected to learn should include a clear focus on thoroughly understanding the content of K–12 mathematics and science, rather than only looking forward to the study of more advanced topics. Similarly, Usiskin (2001) recommended that teacher education should attend to advanced topics that will help teachers look back at the K–12 content they have studied, and new topics they will teach, with increased understanding, rather than addressing only topics that set a foundation for graduate-level study. Cuoco (2001, p. 170) made a comparable recommendation, but approached it from the opposite direction, suggesting that teachers should have experiences "'mining' the topics they teach for substantial mathematics" in order to develop the knowledge and disposition to understand "the advanced mathematics that ties them together."

Wu (1999) further recommended that teachers develop an understanding of advanced topics which build on topics studied in K–12 mathematics. Ferrini-Mundy and colleagues (2005, p. 28) describe the need for teachers to understand content trajectories in mathematics, which they define as "understanding both the origins and extensions of core concepts and procedures - knowing the basis for ideas in the domain, and understanding how those ideas grow and become

more abstract or elaborated." Although written in the context of mathematics, these needs appear equally applicable in science.

Knowledge of Profound Disciplinary Content

The term "profound knowledge" comes from Liping Ma's (1999) work in which she defined "profound understanding of fundamental mathematics" as "an understanding of the terrain of fundamental mathematics that is deep, broad, and thorough" (p. 120). Those who stress the importance of profound knowledge are primarily concerned with content issues that arise in teaching practice, arguing that the instructional decision-making teachers do when planning, carrying out, and reflecting on lessons depends on their ability to use mathematical or scientific knowledge. When teachers choose tasks to assign, ask questions of students, interpret students' responses, and assess students' understanding, they employ content knowledge differently than academic mathematicians and scientists, or those working in applied fields, use their content knowledge. Ma (1999) described how teachers organize "knowledge packages" of closely related ideas that they use to think about instruction and student learning. These knowledge packages consist of "decompressed" knowledge of content, with topics broken down into very specific, connected, key understandings that can guide interpretations of student thinking and instructional decisions.

Understanding an idea with depth includes connecting it to more conceptually-powerful ideas which form the foundation not only for the idea at hand, but for many other ideas as well; these foundational ideas form the substantive structure of the discipline itself. Ball (1989) suggested that deep understanding provides a basis for (1) establishing the correctness of ideas, (2) giving meaning to ideas, and (3) connecting ideas. Similarly, when discussing the preparation of physics teachers, McDermott, Heron, and Shaffer (2005) wrote:

> Teachers should be given the time and guidance necessary to develop concepts in depth and to construct a coherent conceptual framework. They need to be able to formulate and apply operational definitions so that they can recognize precisely and unambiguously how concepts differ from one another and how they are related. (p. 20)

The Conference Board of the Mathematical Sciences (2001) recommended a focus on deep understanding for the preparation of teachers, particularly the "need to understand the fundamental principles that underlie school mathematics," further recommending that "[a]ttention to the broad and flexible applicability of basic ideas is preferable to superficial coverage of many topics." Kennedy (1997) identified the knowledge to distinguish these central ideas in the discipline from minutiae as a central aspect of conceptual understanding of a discipline. In science, the physics education community has led the field in designing courses for prospective teachers consistent with these recommendations, including Physics by Inquiry (McDermott, 1996) and Physics for Elementary Teachers (Goldberg, 2006). In contrast to traditional survey courses, these physics courses focus on a small number of topics in order to develop deep understanding of the fundamental ideas within those topics.

Understanding an idea with breadth entails both connecting it to ideas at a similar conceptual level, and identifying examples that illustrate the idea (Ma, 1999). Kennedy (1997) and McDermott, Heron, and Shaffer (2005) similarly describe conceptual knowledge of an idea as including both an elaborated understanding of the idea itself, and a sense of its relationships to

other ideas. Understanding relationships among ideas includes knowledge such as how the same idea can underlie different procedures or can be represented in different ways for different purposes.

To understand ideas thoroughly is to weave them together in ways that facilitate navigation from one idea to another, bringing coherence to what might otherwise appear to be an unrelated set of ideas (Ma, 1999). The Conference Board of the Mathematical Sciences (2001) recommended that teachers "learn mathematics in a coherent fashion that emphasizes the interconnections among theory, procedures, and applications." Such coherent knowledge provides pathways to and from key foundational ideas, and across topics.

Ways of Knowing in the Discipline

Mathematics and science are more than simply accumulated bodies of knowledge; each embodies a unique set of ideas about what it means "to know" something and about how that knowledge is generated, sometimes referred to as the syntactic structure of the discipline. The NSTA (2003) urged explicit attention to science as a way of knowing in teacher preparation programs. Similarly, the Conference Board for the Mathematical Sciences (2001) recommended that teacher preparation include a specific focus on the nature of knowing and generating knowledge in the discipline of mathematics. An understanding of ways of knowing in mathematics and science has two aspects: (1) how knowledge is formally established in the disciplines; and (2) habits of working and thinking that characterize the disciplines.

How Knowledge is Formally Established in the Disciplines

Science and mathematics each embody a unique collection of ideas about how knowledge generation arises and accrues within the discipline. The recommendations from the Conference Board of the Mathematical Sciences (2001), and related arguments for this kind of teacher knowledge of both mathematics and science, include attention to the formal establishment of knowledge in the disciplines through mathematical proof and scientific inquiry.

Cuoco (2001, 2003) suggested that a teacher needs to know mathematics "as a mathematician," including the nature of generating knowledge in the discipline. He noted, for example, the importance of understanding that doing mathematics often involves temporarily accepting certain ideas as true without proof to determine whether they offer productive avenues of thought, later seeking proofs for these ideas if they turn out to be helpful.

In their studies of the work of teaching, Ball and colleagues have also identified ways of knowing and working in the discipline as important aspects of teachers' mathematical knowledge. Formal ways of presenting knowledge are a part of this knowledge base: "Teaching also involves using tools and skills for reasoning about mathematical ideas, representations, and solutions, as well as knowing what constitutes adequate proof" (Ball, 2003, p. 6-7).

In science, the ways of knowing may vary among disciplines. While the same general epistemological principles hold across disciplines, some phenomena (e.g., chemical reactions) lend themselves to classical experimentation, while others (e.g., the birth and death of stars) do not. Given these varied methods of inquiry, teachers' attempts to impart "the scientific method" to students may do more harm than good. Referring to the preparation of science teachers, NSTA (2003) stated:

> In general, the term "scientific method" (for the hypothetico-deductive method) should be avoided, since it may lead students to believe there is only one way to conduct scientific inquiries. Inductive studies have played a valuable role in science, as have mathematical and computer modeling. Hypotheses are not used formally by scientists in all research, nor are experiments per se the substance of all research. Candidates should study cases in which different approaches to inquiry are used in science, and should endeavor to communicate such differences to their students. (p. 19)

Habits of Working and Thinking that Characterize the Disciplines

In describing the successful science teacher, NSTA (2003) stated:

> Teachers of science engage students effectively in studies of the history, philosophy, and practice of science. They enable students to distinguish science from non-science, understand the evolution and practice of science as a human endeavor, and critically analyze assertions made in the name of science. (p. 16)

Similarly, the Conference Board of the Mathematical Sciences (2001, p. 8) has recommended that teacher preparation include a specific focus on the nature of knowing and generating knowledge in the discipline of mathematics. The habits of working and thinking that characterize the disciplines and underlie the generation of knowledge are included alongside the formal means of establishing knowledge (i.e., mathematical proof, scientific inquiry). In addition, the Board recommended that teacher preparation include, "[a]ttention to the broad and flexible applicability of basic modes of reasoning" and "should develop the habits of mind of a mathematical thinker." These habits of mind include "actions like representing, experimenting, modeling, classifying, visualizing, computing, and proving." Wu (1999) provided more elaboration:

> Content knowledge of mathematics includes the knowledge of how mathematics is usually done: the unending trials and errors, the need to search for concrete examples and counterexamples to guide one's intuition, and the need to make wild guesses as well as subject these guesses to logical scrutiny. (p. 3)

Cuoco (2001, p. 171), too, noted the importance of teachers understanding "how mathematical results are obtained" as opposed to just "how they are presented." He described the underlying thinking and work of mathematics as including "false starts, extensive calculations, experiments, and special cases." Cuoco (2003) termed this kind of knowledge as knowing mathematics "as a philosopher," including the habits of mind that characterize mathematical work.

The American Association for the Advancement of Science's *Science for All Americans* (1990, p. 200) asserted that "[s]cience, mathematics, and technology are defined as much by what they do and how they do it as they are by the results they achieve," and recommended that teachers provide students with "some experience with the kinds of thought and action that are typical of those fields." Similarly, the National Research Council's publication *Ready, Set, SCIENCE!* (Michaels, Shouse, & Schweingruber, 2008, p.19) includes "generating scientific evidence" as an important component of effective science instruction, suggesting that teachers need to highlight that "[e]vidence is at the heart of scientific practice," and that "proficiency in science entails

generating and evaluating evidence as part of building and refining models and explanations of the natural world."

Pedagogical Content Knowledge

Teachers clearly need to understand mathematics/science content in order to teach it. However, the fact that not everyone who has deep content knowledge is an effective teacher makes it equally clear that content knowledge alone, or in conjunction with an understanding of ways of knowing in the discipline, is not sufficient. Effective mathematics and science teaching also requires understanding of content-specific knowledge that is uniquely helpful in the work of teaching, which Shulman (1986, 1987) originally conceptualized as pedagogical content knowledge. The idea of pedagogical content knowledge has been expanded both by elaborating on and adding to the aspects of this knowledge Shulman first identified, particularly by describing content-specific ideas in relation to the work of teaching. Brodie (2004) characterized teachers' mathematical content knowledge as involving both the mathematical practices and the teaching practices in which teachers engage as a part of their work. Magnusson, Krajcik, and Borko (1999) described domains of pedagogical knowledge that are specific to science disciplines, for example, knowledge of areas of student difficulty related to particular science concepts.

Studies of teaching have raised another important distinction about pedagogical content knowledge as it was originally described by Shulman. Pedagogical content knowledge does not only apply to specific content-related situations that can be predicted; teachers also need broad and flexible knowledge of how students think about content upon which to base instructional decisions that arise in unanticipated ways (Ball, 2002; Ball & Bass, 2000; Ball, Lubienski, & Mewborn, 2001).

Three aspects of pedagogical content knowledge can be used to characterize these views on what teachers need to know in this area: (1) knowledge of student thinking, (2) knowledge of implications for instruction; and (3) knowledge of curriculum.

Knowledge of Student Thinking

A key area of pedagogical content knowledge is knowledge of students' thinking in mathematics and science. Three general and related aspects of knowledge of students' thinking have been identified as important to the work of teaching science and mathematics. First, teachers' knowledge might include understanding which ideas are prerequisites or foundations for more sophisticated understandings (e.g., AAAS, 2001). Second, teachers might know the ways that students typically think about ideas, ideally from research that has identified informal or intuitive ways that students commonly approach problems involving specific content ideas. For some ideas, students' informal or intuitive thinking may be very close to correct understandings; for other ideas, students' prior experiences may result in initial conceptions that are counter to established disciplinary understandings. Third, teachers might understand how understanding develops from less sophisticated to more sophisticated ideas (e.g., Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Catley, Lehrer, & Reiser, 2004; Lehrer & Schauble, 2000). Knowledge of such cognitive development of ideas offers teachers frameworks for guiding students' growing understandings of specific concepts.

Knowledge of Implications for Instruction

As research has identified both the importance of student prior conceptions, and the ways students are likely to think about particular concepts, professional development has increasingly focused on helping teachers understand student thinking. But just as content knowledge is necessary but not sufficient for effective teaching, understanding of student thinking addresses only part of the challenge. Teachers need also to understand how particular instructional experiences can build on students' existing knowledge to provide opportunities for them to learn specific mathematics or science ideas.

Ferrini-Mundy and colleagues (2005) describe an area of teacher content knowledge labeled "applications and context," suggesting that teacher knowledge includes understanding situations and circumstances in which particular content ideas arise. These situations could be purely within the discipline (e.g., in science, while studying catalysis in a chemistry class, looking at how enzymes function as catalysts in cells; or in mathematics, examining the ratios and proportional relationships that arise in similar geometric figures) or could come from applications outside of the discipline (e.g., in science, examining the construction of different kinds of bridges in the study of balanced forces; or in mathematics, creating mathematical models from data to optimize pricing for a cookie sale). This type of knowledge is considered a part of pedagogical content knowledge because teachers' knowledge should take into account how instructional activities that rely on applications will help students learn important content ideas and generalize them from the specific context in which they were learned, not just how they can demonstrate that content ideas are useful in the "real world" (NCTM, 2000). As is the case with representations, models, and analogies, teachers need to be able to evaluate applications for their affordances and limitations.

Understanding representations of mathematics or science concepts and how they might be used in instruction is an example of this type of knowledge (Ferrini-Mundy et al., 2005; Magnusson, Krajcik, & Borko, 1999; Usiskin, 2001). Teachers' knowledge of representations includes understanding both the conceptual integrity of each representation and its comprehensibility to an audience of learners (Kennedy, 1997). Knowledge of instructional activities is another example in this area. Content-specific knowledge of activities attends to what aspects of the content being addressed are highlighted in a given activity, and which might be obscured (Magnusson, Krajcik, & Borko, 1999). Such knowledge supports understanding of what aspects of a targeted concept can and cannot be addressed well with a particular instructional activity.

Knowledge of Curriculum

State and national content standards describe the goals of mathematics and science education, sometimes for individual grades or courses, and other times for grade bands such as 6-8. Although targeting the same sets of standards, instructional materials developers make different decisions about the relative emphasis to devote to particular topics, the sequence with which to address topics, and how to engage students with the mathematics/science ideas. Deepening teachers' pedagogical content knowledge when they are learning to use specific instructional materials involves helping them understand how the materials organize the mathematics or science content for classroom teaching. This knowledge includes understanding how content ideas are sequenced (which ideas are introduced earlier that are used as the foundation for learning other ideas later), how connections are made (which ideas are tied together and in what ways), and how the various activities and their sequencing in the instructional materials are

intended to contribute to mathematics/science learning goals. Teachers need opportunities to consider how and why the materials are designed the way they are, especially if the instructional materials are not explicit about the "storyline" of the various activities,.

Balancing and Sequencing the Facets of Teacher Content Knowledge: Different Perspectives in Professional Development

Most science and mathematics professional development providers would agree that all of the facets of teachers' content knowledge are important for teaching. If the time and resources for professional development were unlimited, professional development programs would be able to develop teacher knowledge of all of these facets of content knowledge. Given that there is not enough time to do it all, designers of professional development need to make trade-off decisions.

Often these decisions are based on beliefs about what knowledge is most important for teachers. For example, one perspective is that teachers can't teach what they don't know. Therefore, it is important to start with mathematics/science content, and only after teachers themselves have a sufficiently deep understanding of the content, move to considering classroom application. Another perspective is that teachers are by their very nature practitioners and that professional development should start with classroom applications, e.g., trying to analyze student work, which will provide a context for engaging the teachers in learning mathematics/science content. Also factored into these trade-off decisions are other considerations such as the needs of the teachers in relation to the selected content areas, how much time is available for the particular professional development program, and the knowledge and skills of the individuals available to implement the professional development.

THUMBNAILS OF THE CASE PERSPECTIVES

The stories in this volume are narratives from MSP projects, each describing the professional development the project provided to deepen teacher content knowledge. The intent of these narratives is to provide rich illustrations of how professional development programs have approached deepening teachers' content knowledge—which facets they chose to address, how the facets were balanced and sequenced, and what strategies were used to develop teacher understanding of the different facet areas.

Many of these projects included strategies designed to address systemic factors in addition to the professional development they provided to teachers. Some worked directly with classroom teachers, while others focused their professional development efforts on teachers designated as teacher leaders, and the teacher leaders went on to lead professional development with other teachers in their districts. Projects varied in the grade-range of teachers they targeted, with some including teachers from many grade levels and others focusing on a narrow grade-range. All of them provided professional development intended to deepen teacher content knowledge in mathematics, science, or both. In each narrative, the author has focused on a single professional development offering, generally comprising 30 to 60 contact hours, in many cases focused on a particular topic in science or mathematics.

Below, we provide brief sketches of the facets addressed in the professional development offerings depicted in the narratives, although projects may have emphasized different facets in other professional development activities. Together, these sketches illustrate the fact that educators may take a wide range of perspectives when designing and implementing professional development to deepen teacher content knowledge.

Indiana Mathematics Initiative Partnership

The Indiana Mathematics Initiative Partnership project aimed to deepen elementary and secondary teachers' disciplinary and pedagogical content knowledge through the examination and implementation of the district's mathematics curriculum programs. The goals of the professional development activities for the elementary component were to support teachers' implementation of the standards-based curriculum, *Everyday Mathematics*; improve their mathematics content knowledge at the student-level; and promote and develop their abilities as teacher leaders in their own schools and districts. The case focuses on the professional development experience of one cohort of elementary teacher participants.

Milwaukee Mathematics Partnership

The Milwaukee Mathematics Partnership's central aim was to deepen teacher leaders' mathematics content knowledge by actively engaging them in learning experiences around advanced mathematics content; modeling how knowledge is developed in the field of mathematics; and emphasizing the importance of mathematical reasoning and justification. The project also sought to increase teachers' pedagogical content knowledge, specifically their ability to recognize and develop students' mathematical thinking. This case provides a description of two specific content-focused sessions for teacher leaders around the study of algebra and algebraic relationships.

Southwest Pennsylvania Math Science Partnership

The Southwest Pennsylvania Math Science Partnership focused on preparing elementary and secondary mathematics and science teacher leaders to facilitate high-quality professional development in these subjects. The case describes the professional development experience for one cohort of elementary science teacher leaders. Participants investigated advanced disciplinary content in sessions which modeled effective science instruction, including how knowledge is generated in the disciplines. These content experiences served as springboards to examine instructional decision-making and student thinking and learning. School year sessions focused on formative classroom assessment to support student learning.

Arizona Teachers Institute

The Arizona Teachers Institute project sought to deepen middle-grades teachers' disciplinary knowledge at an advanced level, including connections between the advanced knowledge and student-level knowledge, as part of a degree in mathematics teacher leadership. The project also addressed participants' pedagogical content knowledge and their understanding of logic and proof as mathematical ways of knowing through discussions centered on the disciplinary content topics. The case describes the first course in the Institute's mathematics content sequence, Numbers and Number Sense.

Michigan Teaching Excellence Program
The Michigan Teaching Excellence Program aimed at deepening middle school teachers' content knowledge and understanding of the nature of scientific inquiry by involving them in authentic research experiences. The project engaged teachers in field studies, supplemented by lectures, related to the Michigan state Earth Science curriculum and utilizing selected local and state-wide geographical features. Field study experiences modeled the nature of science, focusing on the centrality of evidence, careful documentation, and reflection on emerging understandings of science concepts. The case describes the professional development teachers experienced in Earth History.

Life Sciences for a Global Community Teacher Institute
The Life Sciences for a Global Community Teacher Institute was focused on deepening high school Biology teachers' understanding of science content knowledge and applying that knowledge to their instructional practice. Master's degree content courses were offered that actively engaged teachers in the process of science research to deepen both their content knowledge and their understanding of the nature of science. An academic year distance-learning program extended content learning from the summer into the academic year and facilitated transfer to practice. The case provides a description of one of the biological content courses: Evolutionary Ecology.

Greater Birmingham Mathematics Partnership
The Greater Birmingham Mathematics Partnership project worked to deepen K–20 teachers' disciplinary content knowledge in an inquiry-based learning environment based on constructivist learning principles. The project's summer mathematics courses included attention to pedagogical content knowledge through readings and discussions, and engaged participants in generalization and justification as mathematical ways of learning and knowing. Professional development activities during the academic year focused on teacher leader development. The case describes participants' experiences in the foundational mathematics course, Patterns, in which they explored a variety of mathematical patterns using algebraic and geometric representations.

North Cascades and Olympic Science Partnership
The North Cascades and Olympic Science Partnership project aimed to deepen the science content knowledge and pedagogical content knowledge of teacher leaders in grades 3-10. Participants were engaged in content immersions intended to help them develop scientifically-accurate understandings of relevant big ideas in science; the roles of evidence and questioning as scientific ways of building knowledge were modeled throughout these and other professional development activities. Facilitators also modeled the instructional strategies teachers were expected to use in their classrooms, and some assignments directly addressed pedagogical knowledge goals. The case describes how the project incorporated principles of learning theory as an organizing framework for the professional development program.

Oregon Mathematics Leadership Institute
The Oregon Mathematics Leadership Institute focused on developing K–12 teachers' disciplinary and pedagogical content knowledge through deep investigation of the mathematics of tasks that might be used in K-12 mathematics classrooms. In addition to mathematics content, the courses were designed with specific attention to socio-mathematical norms, addressing issues

of status differences among learners, and the selection and implementation of "group-worthy" tasks for group work; teachers also participated in an annual collegial leadership course. Facilitation of Institute courses was intended to model a push for generalization and justification as mathematical ways of learning and knowing. The case provides descriptions of several mathematical tasks participants engage in during Data and Chance, one of the first-year mathematics courses.

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