Perspectives on Deepening Teachers' Mathematics Content Knowledge: The Case of the Milwaukee Mathematics Partnership

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Mathematical Knowledge for Leading: A Journey into Deepening Teacher Leaders' Understanding of Algebra and Algebraic Relationships

Abstract

The *Milwaukee Mathematics Partnership* (MMP) is a collaboration among the University of Wisconsin-Milwaukee, Milwaukee Public Schools, and Milwaukee Area Technical College. A major component of MMP has been the development of a cadre of approximately 130 elementary and middle grades teacher leaders committed to leading improvements of their schools' mathematics programs while maintaining full-time teaching responsibilities. For each of five years, MMP provided a series of monthly professional development sessions targeting a selected mathematics content strand (e.g., algebra and algebraic reasoning, measurement and geometric reasoning). Through activities designed and facilitated by teams of university mathematics faculty, mathematics educators, and teachers-in-residence, the teacher leaders considered mathematics content in terms of "big ideas." For example, the algebra strand included five big ideas: equivalence, variable, patterns, linearity, and properties. Mathematics content knowledge was addressed from four perspectives to situate it within various aspects of the work of teacher leaders: (1) mathematical knowledge held by oneself, (2) mathematical knowledge held by other teachers, (3) mathematical knowledge held by students, and (4) the mathematical knowledge given emphasis in curriculum and assessments, specifically the Wisconsin Model Academic Standards and the Wisconsin Assessment Framework.

Introduction

A long history of collaboration exists between the Milwaukee Public Schools and the University of Wisconsin–Milwaukee for mathematic education. These partnerships have focused on both smaller teacher professional development projects funded through state grants, such as the Eisenhower Professional Development program, as well as through large-scale efforts including the College Board Equity 2000 program and the National Science Foundation's Urban Systemic Initiative. The establishment in 1999 of the Milwaukee Partnership Academy (PK-12), an urban PK-16 council, provided the opportunity to further strengthen the university-district partnership and to expand it to include the local community college and the teachers' union. As an initiative of this council, the *Milwaukee Mathematics Partnership* (MMP) became a unified effort to build the capacity of schools for the continuous improvement of student success with challenging mathematics.

A major focus of the MMP has been to develop teachers' mathematics content knowledge. Central to our strategies was the establishment and ongoing mathematical and leadership development of school-based mathematics teacher leaders. We focus our story on the development of elementary teacher leaders' content knowledge in the third year of the Partnership. The following comments by two of the teacher leaders reflect the value they attributed to the deepening of their content knowledge and the importance it held to their effectiveness as leaders.

> *As I develop my understanding of math content, especially the knowledge that reflects the focus of our MMP trainings, I find I am better able to field questions, provide strategies, and move my staff in a direction that reflects the core ideas and goals of the MMP.*

My understanding of math content grew by the sessions devoted to algebraic reasoning across the grade levels. This became one of the hallmarks of my conversations with teachers and it also led to an emphasis on cross-grade level sharing of math concepts.

As a preview to our story, consider the three mathematical tasks shown in Figure 1. How would you respond to each task? How might students respond? What big mathematical ideas are associated with each task? Are these tasks similar to those in curriculum materials or on assessments? How might you use these tasks with teachers to deepen their algebraic reasoning? These tasks prompted the teacher leaders to reflect on their own mathematical knowledge, in particular their understanding of the principles and properties of mathematics. Then each became as a key component of a professional development session on deepening participant understanding of algebra and algebraic relationships.

Task 1. Explain and justify your strategy for finding the solution to

$$48 + 24 = \square + 27$$

Task 2. Explain how you know whether the following statements are true or false

$$13 \times 9 = 90 + 27$$

$$13 \times 9 = 130 - 13$$

Task 3. Describe a story situation or context that could be modeled by

$$F = 10(S - 55) + 40.$$

Figure 1. Three Prompts to Launch Reflection and Discussion

The story begins with a description of the Partnership and our common vision for challenging mathematics. Next we provide background on our mathematics teacher leader model and our theory of action for the content development of teacher leaders. Then we delve into the year-long professional learning of the mathematics teacher leaders as they studied algebra and algebraic relationships. The story closes with discussion of the impact on their mathematical knowledge and leadership practice.

The Partnership

The *Milwaukee Mathematics Partnership* (MMP) is a unique collaboration among a large urban district, a four-year urban university, and a two-year technical college. The partnership includes University of Wisconsin-Milwaukee (UWM), Milwaukee Public Schools (MPS), and Milwaukee Area Technical College (MATC). Our work began in 2003 centered on improving student learning of mathematics across the Milwaukee Public Schools, the largest district in Wisconsin and the 30th largest district in the nation.

The University serves as the lead partner with strong commitment of mathematics faculty and mathematics education faculty. UWM is a large urban-research university, as well as the major provider of teachers for the Milwaukee school district. MATC, the most diverse college in the state and the Midwest's largest technical college, brings additional mathematics faculty into the work of the partnership. Both UWM and MATC are the post-secondary education destinations for many of the school district's graduates.

The high-poverty school district, has an enrollment of approximately 83,000 students K-12 in nearly 200 schools, with 81 percent of students qualifying for free or reduced priced lunch. The district is diverse, with 57 percent of students African-American, 23 percent Hispanic, 12 percent White, 5 percent Asian, and 1 percent Native American. The percent of students with special education needs and those that are English Language Learners has grown each year of our partnership, representing 19 percent and 10 percent of students, respectively, in 2009.

Given the challenges of a large urban district, it is notable that the MMP with five years of intense work has seen steady and significant improvement in mathematics achievement for students throughout the district. Of particular note are the gains revealed on the 2008–2009 state test compared to the previous year. The percentage of students scoring at or above the proficient level in mathematics increased in all grades tested (grades 3-8 and 10) with an average increase of more than five percentage points. Grades 4 and 8 students posted the largest gains at nine and 10 points respectively. In addition, achievement gaps have narrowed for major subgroups of students within the district, except for students with special education needs.

Framework and Vision of Challenging Mathematics

To establish a common vision and understanding of challenging mathematics across the Partnership, we developed the mathematics framework shown in Figure 2 as our definition of student proficiency in mathematics. We have remained focused on this vision for the teaching and learning of mathematics throughout all the years of the partnership. The framework includes the five components of mathematical proficiency— understanding, computing, reasoning, applying, and engaging—as presented in the National Research Council's report *Helping Children Learn Mathematics* (NRC, 2002) and drawn from the report *Adding It Up (*NRC, 2001). These five components revolve around the Wisconsin content standards of number, algebra, statistics, probability, geometry, measurement, and their interconnections. Our goal has always been that this vision will drive classroom practice, define high-quality teaching of challenging mathematics, and be incorporated into the entire teacher learning continuum from teacher preparation to continuing professional development to teacher leadership.

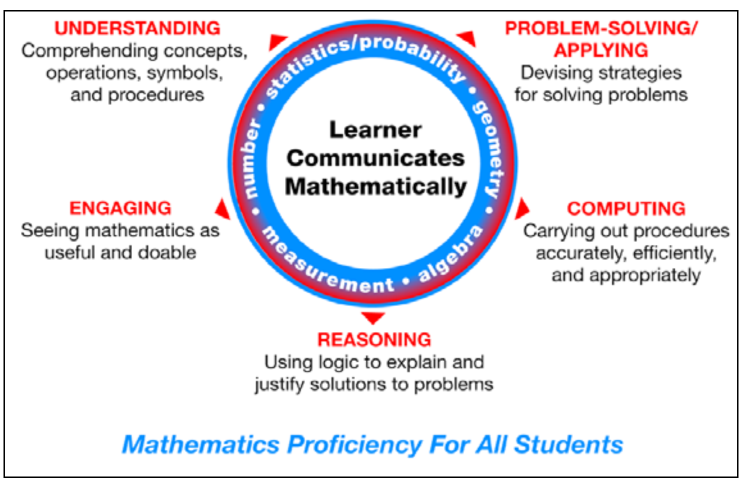

Figure 2. MMP Mathematics Framework

Within mathematics, we selected a subset of content strands for concentrated study each year by our cadre of teacher leaders while continuing to address the proficiency components of the framework. In addition to providing the focus for our work in deepening the content knowledge of the teacher leaders, the strands also became a unifying theme for further learning by teachers throughout the district. For example, as the teacher leaders studied algebra, so did “the district,” as numerous professional learning opportunities were reflective of the direction set in the teacher leader seminars. The strands of study were: mathematical processes in Year 1, rational numbers and operations with fractions in Year 2, algebraic relationships in Year 3, measurement and geometric reasoning in Year 4, and probability and statistical reasoning in Year 5.

Mathematics Teacher Leaders

Our mathematics teacher leader model was based on the willingness of full-time classroom teachers to step forward and provide mathematics leadership for their schools. This position did not exist prior to the Partnership. The principal of each school, with input from the school’s learning team, identified an individual among their current staff for the position. The learning team comprises the principal, literacy coach, mathematics teacher leader, and other key school personnel. The team meets regularly to provide instructional leadership by monitoring student academic achievement and developing school-wide priorities and strategies. This process resulted in a cadre of approximately 130 mathematics teacher leaders at the elementary and middle school levels. The retention rate has averaged about 72 percent of the teacher leaders continuing from year to year, with about 50 percent serving for three or more years.

From the start, we realized that the Math Teacher Leader alone, as one single person, could not address all the needs for mathematics education within a school. It was our vision and belief that it would be vital to connect the teacher leader to the structure of a learning team and to engage the entire learning team in bringing about change for mathematics within a school. The mathematics teacher leader became the voice for mathematics on the learning teams as a liaison between the school and the district, including informing the learning team of MMP initiatives. The MMP trains the mathematics teacher leader, the mathematics teacher leader informs the school-based learning team, and the learning team engages the entire school staff on issues related to improving the teaching and learning of mathematics and decides how best to utilize the mathematics teacher leader.

Given that the teacher leaders were not released from their teaching responsibilities, the main criterion for selection of a mathematics teacher leader was not depth of mathematical understanding, but rather the willingness of an individual to serve in this position and make a commitment to the Partnership expectations. The MMP recognized that these new leaders needed to deepen their content knowledge in order to become confident in supporting the work of mathematics at their individual school sites. The Partnership provided monthly professional development seminars for the teacher leaders. These seminars comprised three strands—deepening mathematics content knowledge, studying formative assessment principles and practices (Black & Wiliam, 1998; Chappuis, Stiggins, Arter, & Chappuis, 2005), and building leadership skills in collaborating with colleagues.

Content Development for Teacher Leaders

The mathematics content components of the seminars were developed and facilitated by a team that included mathematics faculty, mathematics educators, and teachers-in-residence. The teachers-in-residence were classroom teachers from the district who were selected through an application process. They were given a two-year special assignment at the university to support the work of the MMP. The composition of our team allowed us to draw upon our expertise and experiences to develop content-focused sessions that integrated the rigor of the mathematics and mathematical reasoning with the practice-based needs of teacher leaders (Ball & Cohen, 1999; Smith, 2001). In fact, we believe this connection between mathematical rigor and classroom practice was essential for deepening the mathematics knowledge of our teacher leaders.

As we considered the mathematical knowledge needed for leading, four sites of situated knowledge emerged. The four sites or perspectives, shown in Figure 3, included: (1) mathematical knowledge held by oneself, (2) mathematical knowledge held by other teachers, (3) mathematical knowledge held by students, and (4) the mathematical knowledge given emphasis in curriculum and assessments. In our planning, we struggled with these perspectives. Originally, we thought we were going to focus on only a single perspective, the content knowledge held by oneself. However, we have gained a greater appreciation of how these perspectives all contribute to a deepening of content knowledge, and how building the teacher leaders' capacity for viewing the mathematics situated in their practice from multiple perspectives has made them stronger leaders.

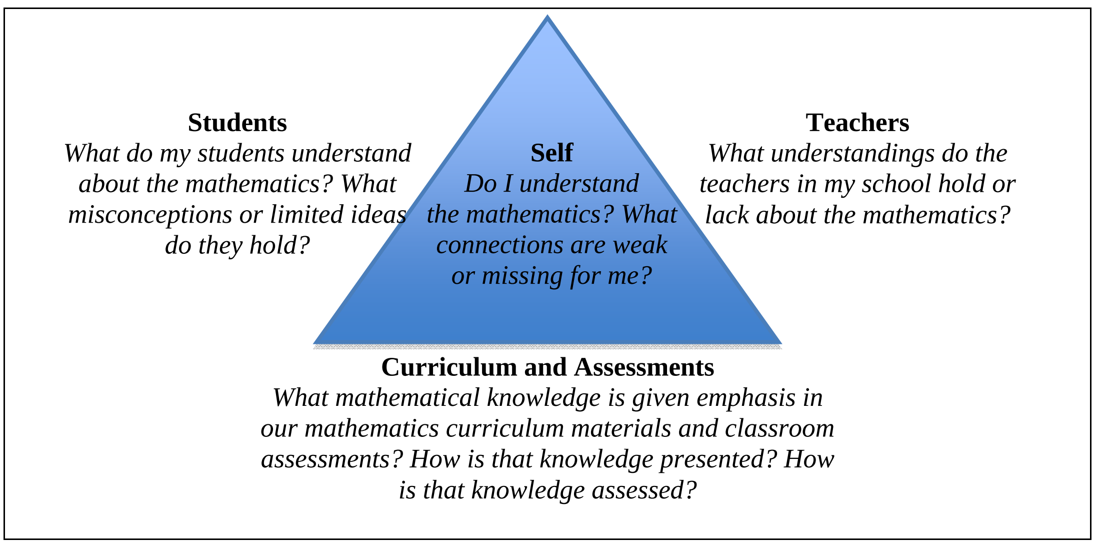

Figure 3. Sites of Mathematical Knowledge for Leading

Our monthly planning meetings prior to the various sessions were always lively as the mathematicians vied for more mathematics, the teachers wanted to keep it engaging and accessible to the leaders as teachers, and the mathematics educators advocated both, drawing on research that offered critical insights into teacher learning and student knowledge acquisition. In general, we would spend hours in discussion, negotiation, and trying out of ideas; then we would draft goals and an agenda for the upcoming session and assign planning tasks. For example, the mathematicians might design examples and exercises that illustrated the mathematical concepts being explored, while the teachers might acquire student work to review during the sessions, and the mathematics educators found relevant passages and information from educational research and literature. Throughout the planning process, we kept in mind our goals of actively engaging teacher leaders in learning important mathematics concepts, how mathematics knowledge is developed, and making connections to their practice.

Deepening Algebraic Reasoning for Leading

In preparation for our collaborative work, the first task was to identify a content strand for the year. We began with an examination of the district's state test scores. The strands most needing improvement at this point in time were algebra and measurement. We chose algebra as a natural continuation and reinforcement of the previous year's emphasis on number and operations. We also thought it would be important for our leaders to view algebra and algebraic reasoning as a PK-12 strand. As Seeley (2004) noted, "A key to success 'in algebra' is the development of algebraic thinking as a cohesive thread in the mathematics curriculum from prekindergarten through high school." Classroom teachers, as well as teacher leaders, often view algebra as something that students do in high school; many elementary teachers do not teach algebraic ideas, even though approximately one-fifth of the state test items at each grade level focus on algebraic thinking.

We then began to pull together resources related to algebra and algebraic reasoning, consistent with district grade-level learning targets and the Wisconsin Standards and Assessment Framework. We used the resources to inform our thinking and then built each of our professional development sessions through discussion and negotiation among mathematicians, mathematics educators, and teachers. Our work was conceptually informed by research on the mathematical knowledge needed for teaching (e.g., Ball, 2003) and research on students' development of algebraic reasoning (Carpenter, Franke, & Levi, 2003; Driscoll, 1999).

Emphasis on Big Mathematical Ideas

The relevant Wisconsin standard is titled, "Algebraic Relationships." This is where we began our discussion. What is algebra? What are algebraic relationships? We thought critically about what students' algebraic reasoning would look like, how that reasoning might develop as the students progressed from elementary to middle to high school, and what teachers would need to know in order to recognize and encourage students' algebraic thinking. The next critical discussion was to identify areas our teacher leaders would study. What do leaders need to know about algebra and algebraic relationships that will support their work with teachers? We had a limited amount of professional development hours to work with our leaders, so a challenge was to reach agreement on the "big ideas" to emphasize throughout the year.

The decision to focus on big ideas rather than topics was pivotal. When teachers understand the big ideas of mathematics, they are more able to "represent mathematics as a coherent and connected enterprise" (NCTM, 2000, p. 17). Yet, our experience has been that few teachers can articulate a big mathematical idea. Charles (2005) suggested that big ideas become the foundation for "one's mathematics content knowledge, for one's teaching practices, and for the mathematics curriculum" (p. 10). We took this one step further and decided that big ideas should also be the foundation for the mathematical development of teacher leaders. This stance situated our work with leaders solidly in mathematics, drew our discussions to a deeper mathematical level, and gave coherence to our work for the year. For example, as we selected topics, activities, and tasks for our professional development sessions we made sure that each was related to, and supportive of, one or more of our big ideas.

The definitions of our big ideas evolved over the course of the year and were adjusted based on our interactions with the teacher leaders and with our own further consideration of the mathematics. Our final list of big mathematical ideas within algebra is shown in Figure 4.

Big Ideas: Equivalence
- Any number, expression, or equation can be represented in different ways that have the same value.
- Mathematical relationships can be represented in equivalent ways—verbally, numerically, visually, and symbolically.

Big Ideas: Variable
- Numbers or other mathematical objects can be represented abstractly using variables.
- Relationships between mathematical objects can often be represented abstractly by combining variables in expressions, equations, or functions.

Big Ideas: Patterns
- Mathematical situations often have numbers or objects that repeat in predictable ways called patterns.
- Patterns can often be generalized using algebraic expressions, equations, or functions.

Big Ideas: Linearity
- A relationship between two quantities is linear if the “rate of change” between the two variables is constant.
- Visually, the graph of a linear relationship will always be a straight line and any straight-line graph represents a linear relationship.

Big Idea: Properties
- For a given set of numbers, there are relationships that are always true, and they are the rules that govern arithmetic and algebra.

Figure 4. Big Ideas Emphasized in Teacher Leader Development

We knew the mathematics teacher leaders were responsible for working with all teachers in their schools, regardless of the grades they taught. Consequently, our intent was to identify big ideas central to the learning of mathematics that allowed for connections and commonalities across grade levels (K-12) and topics. In this way, we hoped the leaders would develop a sense of mathematical coherence and interconnections and would be able to have rich mathematical conversations with teachers at all grade levels.

Professional Development Program Overview

We purposively grounded our work in the Wisconsin Model Academic Standards and the Wisconsin Assessment Framework and revisited it throughout the year. Tying the teacher leaders’ experiences to the expectations for student learning and state testing provided a natural avenue for the leaders to have further conversations at the school level. The sub-skill areas of algebraic relationships in these policy documents aligned well with our focus on big ideas. The three areas included: (1) Expressions, equations, and inequalities, (2) Generalized properties, and (3) Patterns, relations, and functions.

The mathematics teacher leaders met monthly during the 2005-2006 school year, beginning with a two-day kickoff in August just prior to the start of the school year. We had developed nine content sessions for the teacher leaders' journey into algebra and algebraic relationships. The session topics and goals are shown in Table 1. We began with describing patterns and change, then moved to examination of equality and equivalence. Next the focus was on reasoning with relationships and generalized properties. The final area involved expressing linear relationships, as well as some exploration of non-linear relationships. Throughout the planning process, our central aim was to deepen the mathematics teacher leaders' content knowledge in algebraic reasoning by actively engaging them in learning mathematics, modeling how knowledge is developed in the field of mathematics, and emphasizing the importance of mathematical reasoning and justification. We also sought to increase their ability to recognize and develop their students' algebraic thinking, and to provide them with resources they could use to facilitate similar types of learning opportunities with the staff in their schools.

Table 1. Teacher Leader Professional Development Program on Algebraic Relationships

Session		Goals	Key Tasks or Prompts
1	Beginning the Journey into Algebra and Algebraic Thinking	• Link our journey to the Wisconsin Standards and Assessment Framework. • Explore, extend, and generalize patterns. • Begin to examine "big ideas" of algebra.	What is algebra? What is algebraic thinking? Toothpick Bridges Task. Focused reading: Algebra journey (Seeley, 2004).
2	Describing Change	• Analyze and describe "change" in a variety of contexts. • Describe quantitative change with pictures, words, tables, and symbolic rules.	Envelope task: What is changing and how? The Growing Dots Problems. Focused reading, Wisconsin Model Academic Standards on Algebraic Relationships.
3	Equality and the Equals Sign	• Examine the concept of equality through the eyes of mathematics and of students. • Investigate ways to maintain equality in various situations.	Solve 48 + 24 = ❐ + 27 and compare strategies. Envelope task: Analysis of student work samples. Student data on knowledge of the equals sign. Video clips: Equality (Carpenter et al., 2003).
4	Reasoning with Relationships and Properties	• Identify strategies that use computational thinking and that use relational thinking. • Apply reasoning based on relationships and properties to solve problems.	True or False: $13 \times 9 = 90 + 27$; $13 \times 9 = 130 – 13$ Envelope task: Analysis of true and false statements. Video clips: Grade 3 district students reasoning with properties for multiplication facts.
5	Distributive Property Part 1	• Explore generalized properties as connections between arithmetic and algebra.	Partition arrays for whole number context situations, use grid paper, find partial products in arrays, write related equations: 8 x 6, 8 x 15, 23 x 34, 32 x 48. True or False: $25 \times 46 = (20 \times 40) + (5 \times 6)$
6	Distributive Property Part 2	• Make "implicit" reasoning "explicit" as part of algebraic thinking. • Generalize the distributive property as an essential foundation for algebra.	Envelope task: Formulating conjectures. Focused reading: Being explicit (Carpenter et al., 2003); Abstracting from computation (Driscoll, 1999). Partition open arrays, apply the distributive property: 43 x 52, ($4\ ^3/_4$) x ($5\ ^2/_3$), and (4x + 3)(5x + 2).
7	Expressing Relationships	• Develop the algebraic habit of mind of "expressing relationships." • Express relationships with both natural language and with algebraic symbols. • Examine the roles and uses of variables.	Envelope task: Write equations for story problems. Focused reading, Doing algebra (Usiskin, 1997). Describe a context for F = 10(S – 55) + 40. Envelope task: Create stories modeled by equations, and write equations to model story situations. Matching task: Match story situations to equations.
8	Moving Among Representations	• •Make translations among representations (i.e., equations, pictures, words, tables, and graphs) for algebraic situations . • Study linear and non-linear relationships.	Tell a story for the graph: Bathtub Water Level. Jigsaw: Given a story, create a table and graph it. Envelope Task: Create story situations that could be modeled by specific graphs, tables, or equations.

Session		Goals	Key Tasks or Prompts
9	Algebraic Travels: The Journey Never Ends... Using Algebra to Make Decisions	• Represent a story context in multiple ways (table, equations, words, graph). • Use and compare multiple mathematical models to make decisions. • Revisit the "big ideas" of algebra.	"Can You Hear Me Now?" Task. Item analysis: Finding algebra and algebraic relationships in released state test items at all grades. Reflections on algebra learning: Role as a learner of mathematics, a classroom teacher, and a leader.

Professional Development Sessions

Here we provide a walk through of two content-focused sessions. First we describe our opening session that began our study of algebra. Then we describe a session on equality to further illustrate the design and implementation of the professional development program. As we walk through this seminal session on equality, we elaborate on key tasks and prompts, reveal some of our decision-making, include comments from teacher leaders, and incorporate our own commentary on important components of the session. The other sessions had a similar structure and balance of emphasis among mathematical knowledge held by oneself, by teachers, and by students, and the mathematical emphasis in curriculum and assessments.

The Study of Algebra Begins

The professional learning began with the mathematics teacher leaders reflecting on their past experiences with algebra. "What is algebra? What are your memories of learning it?" Each individual reflected silently for a few moments, then used a marker to record phrases, pictures, or diagrams on the paper, graffiti style. Then individuals took turns summarizing the meaning of his or her graffiti. Not surprisingly, many of these elementary and middle school teachers remembered algebra as variables, simplifying expressions, and solving equations, typically devoid of connections to real-world contexts. Some liked algebra, some struggled through it (meaning their high school courses), and others were indifferent.

Next we reviewed the low performance of district students on the state tests for algebraic relationships and noted that approximately 15-20 percent of the points at each grade level are from items on algebraic reasoning. Then we examined the Wisconsin Model Academic Standards and Assessment Framework for expectations of student learning.

Our foray into the mathematics began with emphasis on the big ideas of patterns, equivalence, and variable by using the "toothpick bridges" task. The leaders examined the diagram in Figure 5 and then built bridges with toothpicks. The task was to determine how many rods or "toothpicks" would be needed for a bridge of length 4, length 20, and length 100. The group generalized the observations as expressions. We then compared the expressions with the challenge to visualize and understand how each expression relates to the bridge.

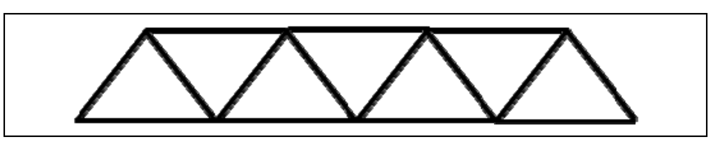

Figure 5. Toothpick Bridge of Length 4

Next the leaders read "A Journey Into Algebraic Thinking" (Seeley, 2004), a message from the president of the National Council of Teachers of Mathematics. The focus question was, "What are characteristics of algebraic thinking to develop throughout grades PK–12?" In small groups, they created lists of three to five key characteristics pulled from the reading. Then as a whole group the leaders reflected on their own experiences in regards to the toothpick bridges task by discussing, "In what ways were you engaged in algebraic thinking today?" This began our year—reflecting on our own learning, investigating mathematics, considering student learning, and pondering some big mathematical ideas.

Equality: The District Studies the Meaning of the Equals Sign

The study of equality by the mathematics teacher leaders ignited discussions on the meaning of the equals sign across the district. We learned how a simple task can be very powerful if the task is carefully selected to bring out students' misconceptions or missing knowledge. The task asks students to determine what number goes in the box for 8 + 4 = ❒ + 7 or for 48 + 24 = ❒ + 27. As Carpenter, Franke, and Levi (2003) noted, "Most elementary students, and many older students as well, do not understand that the equal sign denotes the relation between two equal quantities" (p. 9). The students in our district were no different, which was very surprising to our teacher leaders and the teachers in their schools. It was not surprising to us.

Launching the Mathematics. The session began by asking each teacher leader to individually solve 48 + 24 = ❒ + 27 and to keep track of her or his strategy. We charted a few strategies and compared them. It was highlighted that some strategies involved calculating the sum of 48 + 24 as 72 and then subtracting 27 from this amount to find the missing number. Other strategies used relationships among the numbers in the two expressions that made it unnecessary to actually carry out the calculation. For example, 27 is three more than 24, so the missing number must be three less than 48. This use of relational thinking brought an "aha" moment to some participants while others needed more time to ponder the reasoning. (Since relational thinking was not the focus of this session, we did not consider it further at that point, but made a note to come back to it in a later session.) The participants were asked to reflect on why they approached the task as they did and what their goal was in reaching a solution. In general, the participants shared that they were looking for a number that would make the equation balance or that would result in both sides of the equation having the same value.

Surfacing Student Conceptions: Meaning of the Equals Sign. Next the participants considered three questions: What does the equal sign mean? What would your students say is the meaning of the equals sign? Why do you think your students would answer that way? After jotting down some thoughts individually, the teacher leaders had small group discussions in which they talked through these questions and began to think about a working definition of the equal sign that might make sense to both them and their students. As a whole group debrief, we discussed the surprises that surfaced in their small group discussions. The main surprise was the realization that they had never really talked about the meaning of the equals sign with their students. Most participants thought their students would say "it tells where the answer goes." Another surprise was the struggle to articulate the meaning of the equals sign. One leader asked, "Could we say it means sameness on both sides?" Another summarized, "This is hard to put into words." We then brought into the discussion the related student expectation from the state assessment framework: students will demonstrate an understanding that the equals sign means "the same as."

Envelope Task: Student Work Samples. In preparation for the professional development session, we gathered student work samples on some equality tasks from several different schools across the district. We were overwhelmed as we realized we had gathered far too much student work on far too many tasks than we would be able to use in the session. As we started to look at the student work, we decided to use the two tasks noted above to focus our conversations with the leaders. We selected one task for younger students and one for older students. We compiled the results and selected work samples for analysis by the teacher leaders as an “envelope” task.

A strategy we have found to be successful with students, “envelope” activities were a device we used frequently in our professional development sessions. Project staff developed tasks cards, problem situation cards, and student work sample cards. We then placed the cards in envelopes and gave one to each small group. We found the anticipation of pulling a card out of an envelope as enticing for teachers as it is for students. From a management perspective, it eases assigning roles to group members and encourages interaction and discussion within each group. Sometimes we asked that only one card be pulled out at a time for discussion. Other times each person drew a card and was responsible for leading the discussion related to that particular card. Sometimes the cards needed to be ordered in some way or they were on different colors and required the participants to match or compare them.

For this particular activity, we selected four samples of student work for $8 + 4 = \square + 7$ and put them on blue cardstock and seven samples for $48 + 24 = \square + 27$ and placed them on yellow cardstock. Six of these work samples for the second problem are shown in Figure 6. The small group discussions were followed by a whole group debrief. The directions for the groups were:

Feel free to review both sets of work but get familiar with one. As a table group, work through the following questions:

- *What might account for the variety of answers/responses to the prompt?*
- *How did the students get the answers that they did?*
- *What does this work show us about their understanding?*
- *Do you notice any similarities between the blue set and the yellow set?*

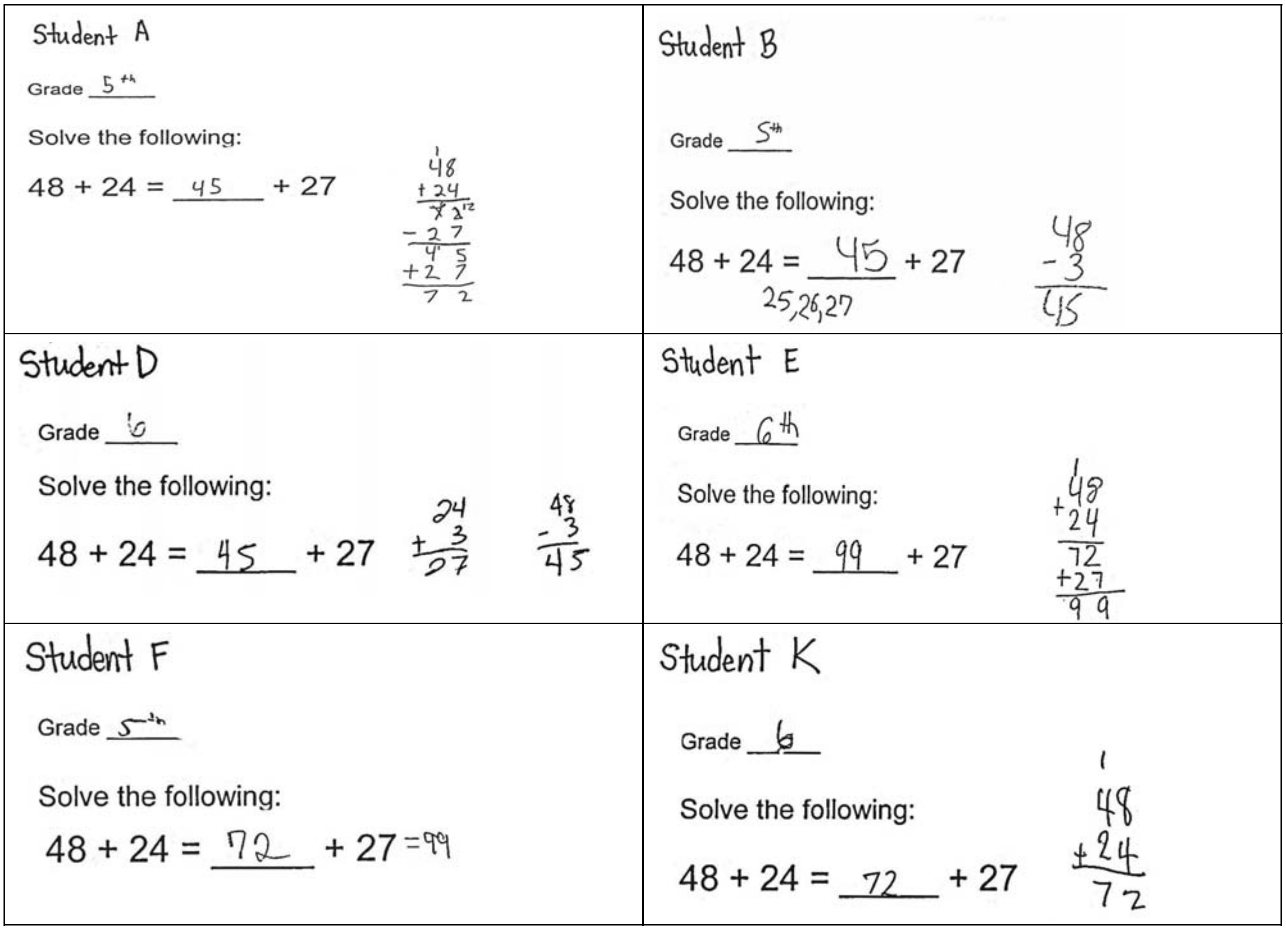

Figure 6. Student Work Samples

Analyzing Student Data: Knowledge of the Equals Sign. Finally, we were ready to reveal the results we had gathered from students in the school district. First we displayed the results for the younger students. The data were from three second grade classrooms. The results showed that only 16 percent of the 43 student work samples had the correct answer of 5 for the task 8 + 4 = ❒ + 7. The most prevalent incorrect answer by 40 percent of the students was 12. Another 12 percent gave an answer of 19 and 2 percent changed the equation to read 8 + 4 = 12 + 7 = 19.

The results for the other task using two-digit numbers are shown in Table 2. While the specific answers are different, the types of reasoning used by these older students were similar to those used by the younger students. Of 176 student work samples, 20 percent gave the correct answer of 45. The most prevalent response by 45 percent of the students was 72 which involved adding the two numbers on the left side of the equals sign, the same as for the younger students. The older students did not just add the three numbers together like the younger students, but were more likely to modify the equation to read 48 + 24 = 72 + 27 = 99.

Table 2. Student Results for 48 + 24 = ❐ + 27

Grade	Number of Students	Answer Given				
		45	72	99	72 & 99	Other
5	50	8	16	2	5	19
6	37	16	15	0	3	3
7	41	6	19	0	9	7
8	48	5	31	0	9	3
Total	176	20%	46%	1%	15%	18%

The results were a bit shocking and eye-opening to the teacher leaders. To lessen the shock, we also showed how the results reflected those found by Carpenter and colleagues: across grades students view the equals sign as an instruction to perform an operation rather than as a sign of equality. It was noted that, "a limited conception of what the equal sign means is one of the major stumbling blocks in learning algebra" (Carpenter et al., 2003, p. 22).

Closing the Session. The final component of the session included viewing a video clip of a kindergarten student solving 5 + 4 = ❐ + 3, reading a brief transcript of a student solving two similar problems, and doing a short reading jigsaw on students' understanding of the equals sign. The session then closed with a discussion of the big mathematical idea of equivalence.

The teacher leaders were given a "homework" or rather "schoolwork" task to try the same two tasks with students in their schools. However, before we could even make the assignment, most were already talking about doing so. The leaders were to try the tasks with at least two different grade levels of students in their schools, summarize the results, and bring the results and a range of student work samples to the next teacher leader seminar. Many teacher leaders went considerably beyond the requirement, reporting using the tasks with students across all grade levels in their schools, and conducting follow-up discussions with teachers at grade level, staff, and learning team meetings. One teacher leader commented,

> *The equal sign task was done by all math teachers in our school and by the learning team members. It opened everyone's eyes to the knowledge of our middle school students and the importance of dialogue amongst students and the teachers.*

We were struck by the power of such a simple task to provoke mathematical discussions across the district because of the window it opened to students' misconceptions and missing knowledge of the equals sign. Another mathematics teacher leader wrote,

> *We posed the same equation, 8 + 4 = ❐ + 7, to a second grade class and a sixth grade class. Both classes had the same percentage of students get the correct answer. The sixth grade teacher was horrified! Many of her students did not know what the equal sign meant. The wrong answer seen the most was 12. This opened her eyes to the fact that she needed to revisit this and other basic algebraic ideas she assumed the students knew already. The second grade teacher realized that she also needed to establish a stronger foundation for understanding the equals sign as meaning a "same as" relation between quantities.*

Impact on Mathematical Knowledge and Leadership Practice

The theoretical perspective driving our work in deepening the mathematics content knowledge of teacher leaders has been that of the mathematical knowledge needed for teaching (Ball, 2003; Ball & Bass, 2003). To study the impact on our professional development program, we administered pretest and posttests using items from the Learning Mathematics for Teaching (LMT) project at The University of Michigan (Ball, Hill, & Bass, 2005; Hill, Rowan, & Ball 2005). The teacher leaders demonstrated significant growth in their content knowledge for teaching algebra after participating in the Partnership's one-year professional development program on algebra and algebraic relationships; with an effect size of 0.30 standard deviations.

The learning at school sites flowed directly from the training of the mathematics teacher leaders during their monthly meetings. As the mathematics teacher leaders were deepening their content knowledge, they were utilizing this learning as part of their leadership practice. As one teacher leader wrote,

> *The MMP has positively impacted my school (teachers and students) in many ways. One way is that they have made it very easy for the Math Teacher Leaders to share information with the rest of their staff. Through their example, modeling, and providing valuable content information, I facilitated an in-service on the importance of the equal sign. MMP made this task easy because they modeled it and provided a script on their website. Feedback from the in-service I facilitated was positive since the activities were focused and meaningful.*

Beyond the more obvious aspects of leadership practice such as conducting a workshop for other teachers, the teacher leaders also reported more subtle yet perhaps more powerful influences of their content knowledge. These subtle changes were becoming apparent in what they noticed in their conversations with other teachers, what they noticed as they visited classrooms or co-taught with other teachers, in their interactions with students, and in the analysis of student work. For example, one leader commented, "I noticed a teacher at my school was teaching that the equal sign means the answer." Another remarked that she had seen teachers stringing together expressions that were not equivalent which prompted her to facilitate a staff discussion on the definition of the equals sign with examples of correct and incorrect use of the equals sign. A third noted that she now asks students questions to draw out relational thinking and no longer just accepts a computational strategy. In general, these and other teacher leaders indicated that they were more aware of the embedding of algebraic learning across grade levels, no longer overlooked the misuses of the equals sign, in particular, and could identify opportunities to further develop algebraic reasoning in students and in teachers. This heightened awareness of algebraic reasoning lead to richer and deeper informal and formal discussions with teachers about mathematics content and about the progression of developing mathematical knowledge across grade levels.

Reflections on the Mathematics Learning of Leaders

Throughout our work, the partnership has strived to build the capacity of schools for continuous improvement in mathematics. When we began our work with the teacher leaders in spring 2004, we met with some initial resistance to studying mathematics held by oneself. This resistance appeared to be two-fold. The leaders seemed uneasy in exposing their lack of or superficial understanding of mathematics content, and they expressed the wish for more practical and immediate strategies to take back for use in their schools. This resistance was short-lived as the "content sessions" soon became the highlight of our monthly teacher leader seminars. Throughout the district, we developed a culture for studying the mathematics and articulating the important mathematical ideas in curriculum materials, daily lessons, and in classroom assessments. Along with this, we developed a culture and expectation for studying the mathematical successes and misconceptions in student work in order to provide appropriate descriptive feedback to students. This work demands thinking hard about the mathematics and being able to articulate the mathematics to oneself, to colleagues, and to students.

We recommend a long-term plan over several years for deepening the content knowledge of mathematics teacher leaders. The approach should allow for connections of the content knowledge to the practice-based work of teacher leaders. We believe it is essential that the instructional team includes mathematics faculty, mathematics educators, and classroom teachers. Each viewpoint was essential in contributing to the success of the program, and the synergy of the three viewpoints led to the development of high quality professional development sessions that could not have been accomplished otherwise.

The identification of a mathematics theme for each year of our Partnership work was beneficial in several ways. First, it allowed us to give focus to an area of mathematics in examining and deepening one's own mathematical knowledge. Second, it gave focus to engaging teachers at school sites in examining the topic throughout the year. Third, it has allowed time to examine the selected content area within the context of practice—curriculum, daily lessons, assessments, and student work. The instructional team planned comprehensive and carefully sequenced content sessions each month to fully use the time allocated. Of course, we still wanted and needed more time each month to further develop the mathematical knowledge of the teacher leaders. However, their learning did not just occur within the content sessions, but rather became embedded into their work as teacher leaders. We continually provided opportunities and connections and even tasks and formats to further support their mathematical work as leaders.

The focus on big mathematical ideas brought coherence to our professional development sessions. It was also an essential component of deepening the teacher leaders' content knowledge. As they grew in their understanding of big ideas, teacher leaders were less likely to see mathematics as a set of disconnected concepts and skills, and more likely to view it as a coherent set of ideas. The leaders also began to translate big ideas into their leadership practice as they considered how Big Ideas connect topics, concepts, and skills across grades. In addition, the focus on big ideas brought coherence to the important mathematical ideas examined through classroom assessments.

The mathematics teacher leaders are building a strong foundation within each school that we believe will be sustained because they see the understanding of mathematics content knowledge at the core of the quality instruction that has, in turn, led to significant gains in student mathematics achievement on our state tests. Our Partnership is strong. The intense and sustained focus on one district has offered us the singular opportunity to create a culture and expectations for the work of teachers and the learning of students in mathematics. We have seen a shift in culture that will not easily be undone, creating widespread and increased demand for continued deepening of teacher content knowledge in mathematics.

References

Ball, D. L. (2003). *What mathematical knowledge is needed for teaching mathematics*? Paper presented at the U.S. Department of Education Secretary's Mathematics Summit (February 6, 2003). Available from http://www.ed.gov/inits/mathscience/ball.html.

Ball, D. L., & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In B. Davis & E. Simmt (Eds.), *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group*, (pp. 3-14). Edmonton, AB: CMESG/GCEDM.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, Fall*, 14-17, 20- 22, 43-46.

Ball, D., & Cohen, D. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In L. Darling-Hammond and G. Sykes (Eds.), *Teaching as a learning profession: Handbook of policy and practice* (pp. 3-32). San Francisco: Jossey-Bass.

Black, P., & Wiliam, D. (1998). Inside the black box: Raising standards through classroom assessment. *Phi Delta Kappan*, 80(2), 139-148.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth, NH: Heinemann.

Chappuis, S., Stiggins, R., Arter, J., & Chappuis, J. (2005). *Assessment for learning: An action guide for school leaders* (2nd ed). Portland, OR: Assessment Training Institute.

Charles, R. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. *Journal of Mathematics Education Leadership, 8*(1), p. 9-24.

Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers, Grades 6-10*. Portsmouth, NH: Heinemann.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal (42)*, 371-406.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Mathematics Learning Study Committee, Center for Education, Division of Behavioral Sciences and Education, National Research Council. Washington, DC: National Academy Press.

National Research Council. (2002). *Helping Children Learn Mathematics*. Mathematics Learning Study Committee, J. Kilpatrick & J. Swafford, Editors. Center for Education, Division of Behavioral Sciences and Education. Washington, DC: National Academy Press.

Seeley, C. L. (2004). A journey in algebraic thinking. *NCTM News Bulletin (September)*. Retrieved January 10, 2010 from http://www.nctm.org/about/message.aspx?id=16837&folderid=3347.

Smith, M. S. (2001). *Practice-based professional development for teachers of mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Usiskin, Z. (1977). Doing algebra in grades K-4. *Teaching Children Mathematics* 3, 346-356.

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