



Nebraska
MATH

UNIVERSITY OF
Nebraska[®]
Lincoln

Mathematics as a Second Language

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Mathematics as a Second Language is

- The first course in the K-5 Master's Program offered by the Vermont Mathematics Institute
- A textbook written by Ken and Herb Gross
- The first course in the Math in the Middle Institute's Master's program for middle level mathematics teachers (5-8)
- A course that focuses on the operations of arithmetic while emphasizing the interrelationships among arithmetic, algebra and geometry. A major theme is understanding algebra and arithmetic through language.



Math in the Middle Instructional Models

SUMMER

- Class meets for one week from 8:00 a.m. - 5:00 p.m.
- 35 teachers – 5 instructors in class at one time.
- Substantial homework each night.
- End-of-Course problem set
 - Purpose – long term retention of knowledge gained.

ACADEMIC YEAR

- Two-day (8:00 – 5:00) on-campus class session.
- Course completed as an on-line, distance education course using Blackboard and Adobe Connect.
 - Major problem sets
 - End-of-Course problem set
 - Substantial support available for teachers



Mathematics as a Second Language

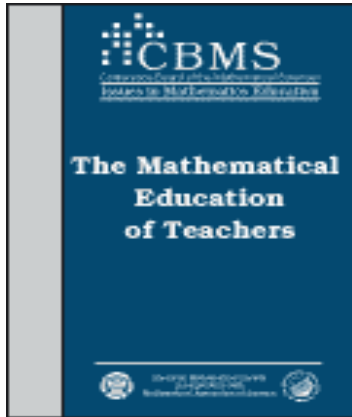
Objectives:

- enhancing mathematical knowledge
- enabling teachers to transfer mathematics they have learned into their classrooms

Audience:

- Course has been taught about 10 times as part of our NSF institute and as a separate professional development course.
- Emphasis has been on middle grade teachers (5-8) but some classes have included high school teachers.





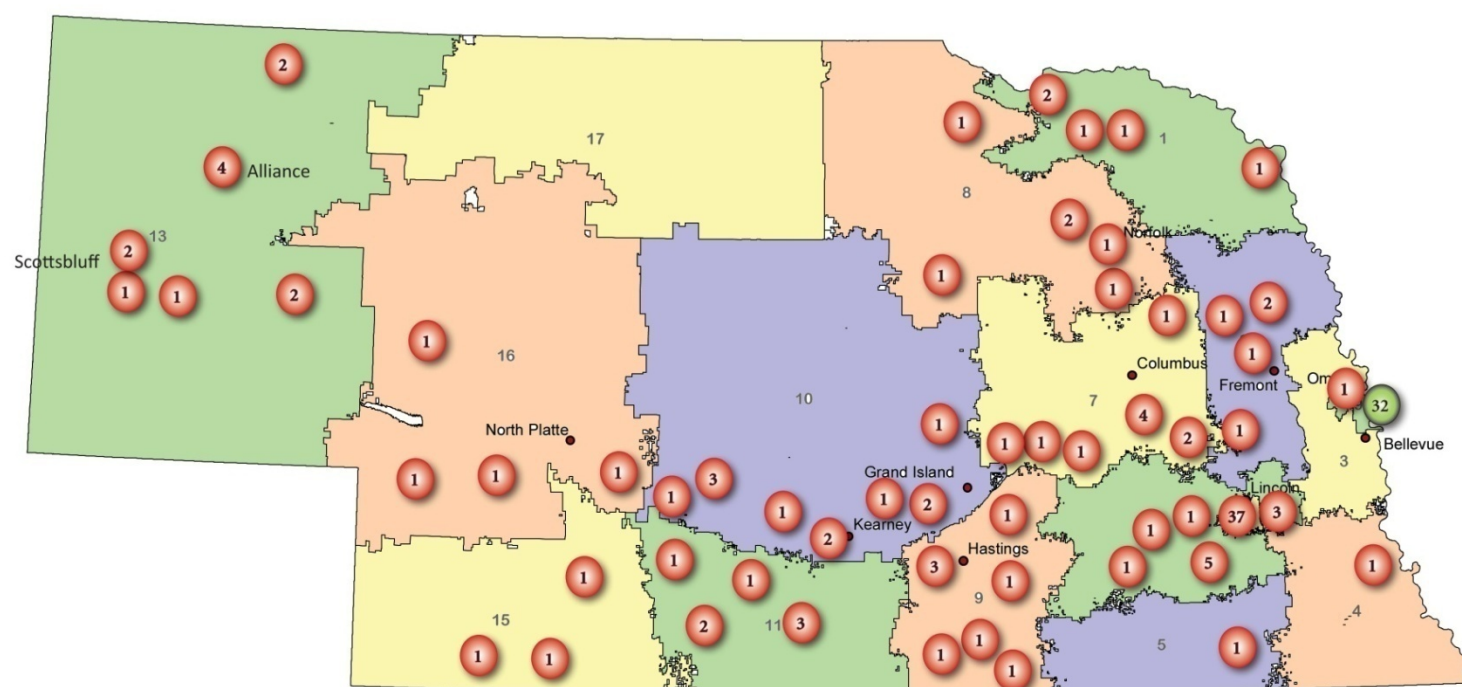
The Mathematical Education of Teachers

Recommendations

- Teachers need mathematics courses that develop a deep understanding of the math they teach.
- Mathematics courses should
 - focus on a thorough development of basic mathematical ideas.
 - develop careful reasoning and mathematical ‘common sense’.
 - develop the habits of mind of a mathematical thinker and demonstrate flexible, interactive styles of teaching.
- The mathematics education of teachers should be based on
 - partnerships between mathematicians, mathematics education faculty and school mathematics teachers.

157 Math in the Middle Teachers

Math in the Middle Teachers by Nebraska Educational Service Units



Typical Cohort	5 th	6 th	7 th	8 th	7-12	HS
32 teachers	7	7	5	7	3	3

The first question:

The Film Problem

There are two photography stores in town that do custom film developing, Perfect Picture (PP) and Dynamic Developers (DD). At PP the cost to develop one roll of specialty film is \$12 but any additional rolls of film cost only \$10. At DD the cost of developing one roll of film is \$24 but each additional roll is developed at a cost of only \$8.

Problem: For what number of rolls of film is the cost of developing the same at PP and DD?

From Mathematics as a Second Language
By Herbert and Kenneth Gross



We encourage teachers to solve the problem in a variety of ways

- Trial and error (adaptive guessing)
- Solution to an algebra problem
- A table of values
- Mental arithmetic
- Graphing two linear equations
- A table of values keeping track of differences



A special emphasis in our MSL course and our Math in the Middle Institute

- The habits of mind of a mathematical thinker

Have you ever noticed that two students (or two adults) appear to know the same “facts” but there is a marked difference in their ability to use that information to answer questions or solve problems?

How do we develop the “habits of mind of a mathematical thinker” in teachers and assist them in cultivating this knowledge among their students?



The Chicken Nugget Conundrum

- *There's a famous fast-food restaurant where you can order chicken nuggets. They come in boxes of various sizes. You can only buy them in a box of 6, a box of 9, or a box of 20. Using these order sizes, you can order, for example, 32 pieces of chicken if you wanted. You'd order a box of 20 and two boxes of 6. Here's the question: What is the largest number of chicken pieces that you cannot order? For example, if you wanted, say 31 of them, could you get 31? No. Is there a larger number of chicken nuggets that you cannot get? And if there is, what number is it? How do you know your answer is correct?*

A complete answer will:

- Choose a whole number "N" that is your answer to the question.*
- Explain why it is not possible to have a combination of "boxes of 6" and "boxes of 9" and "boxes of 20" chicken nuggets that add to exactly N pieces of chicken.*
- Explain why it is possible to have a combination that equals any number larger than N.*



The Triangle Game

(Paul Sally, U. Chicago) Consider an equilateral triangle with points located at each vertex and at each midpoint of a side. The problem uses the set of numbers $\{1, 2, 3, 4, 5, 6\}$. Find a way to put one of the numbers on each point so that the sum of the numbers along any side is equal to the sum of the numbers along each of the two other sides. (Call this an Equal Side Sum.)

- Is it possible to have two different Equal Side Sums?
- Which Equal Side Sums are possible?
- How can you generalize this game?



Mind over Mathematics

Jim Lewis claims to have the ability to read your mind.

Choose a six-digit whole number “n” that repeats the first three digits – e.g. 725725 or 109109 or 226226. Write your number in the following space: My number is: $n =$ _____.

Without knowing what number you choose, Jim can guess a factor of your number (i.e. a number “d” chosen so that n/d is a whole number. (Jim will not guess the number 1 nor will he guess the number n.) After you pick your number, send Jim an email (don’t tell him the number) and he will send you a number d that divides your number.

Your assignment is to expose Jim’s mind reading scam as just good mathematics. On the line below, predict what number Jim will choose as a factor of n. You are permitted a maximum of 7 different choices. Then explain his “mind-reading” trick.



A mathematical thinker with well-developed habits of mind:

1. Understands which tools are appropriate when solving a problem
2. Is flexible in his or her thinking
3. Uses precise mathematical definitions
4. Understands there exist multiple paths to a solution
5. Is able to make connections between what one knows and the problem
6. Knows what information in the problem is crucial to its being solved
7. Is able to develop strategies to solve a problem
8. Is able to explain solutions to others
9. Knows the effectiveness of algorithms within the context of the problem
10. Is persistent in his or her pursuit of a solution
11. Displays self-efficacy while doing problems
12. Engages in meta-cognition by monitoring and reflecting on the processes of conjecturing, reasoning, proving, and problem solving



Pedagogical habits (*sensibilities*):

Frequent ways of acting within a teacher's practice that support the successful doing of the mathematical tasks of teaching and lead to constructive outcomes.



Mathematical Tasks of Teaching

(Ball, Thames & Phelps, 2008)

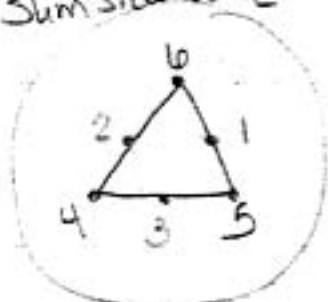
- Responding to students “why” questions
- Giving or evaluating mathematical explanations
- Asking productive mathematical questions



Student A's work on The Triangle Game

The Triangle Game

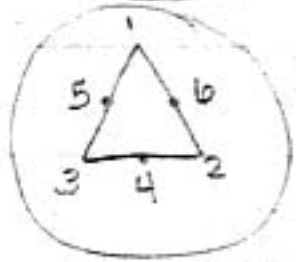
Sum side is 12



11 1
10 2
9 3

1 5 6
2 4 6
3 4 5

Sum side is 9



1 2 6
1 3 5
2 3 4

Student B's work on The Triangle Game

Large outside is 11
Small inside is 10 -
total 21

outside is 10
inside is 11

Generalization - There seems to be a connection between outside + inside numbers. The sum of the outside numbers + sum of inside numbers in each case equals 21 which is the same as adding the 6 numbers. - so I had to find combinations of numbers to equal 21.

9 + 12
10 + 11
8 + 13 won't work because no combinations make 8 + 13.

Student C's work on The Triangle Game

- The text below represents a portion of the work of an eighth grade teacher with secondary certification. Here, Student C offers a justification of the fact that 9 is the smallest Side Sum and 12 is the largest Side Sum.
- *To get the “side sum” with the SMALLEST value for the sums, you would have to put the 3 smallest #s at the vertices. The 3 larger #s would then be put at the midpoints by placing the largest (6) between the smallest (1 & 2), the next largest (5) between the next smallest (1 & 3). That leaves only one place for the 4 to go (between the 2 & 3). This creates a side sum of 9.*
- *To get a larger side sum, reverse that process. Place the 3 largest (4, 5, & 6) at the vertices. Then, the midpoints would be placed with the smallest (1) between the largest pair (6 & 5), then (2) between (6 & 4) and finally the largest of the smaller #s (3) between the smallest sum of the larger #s (5 & 4) to create sums of 12.*



Learning and Teaching Project

Opportunity for teachers to see what is
entailed in translating
mathematical knowledge
into
mathematical knowledge for teaching



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