

# Virginia's MSP Institute: Preparing Mathematics Specialists

Breakout One:  
Selecting Strategies

May 24, 2010

# Functions Algebra Course

- Algebra readiness and beginning algebra concepts for K-8 teachers and Mathematics Specialists who will be coaching K-8 teachers.
- A three credit graduate level mathematics course, typically meeting for 55 contact hours.
- Russell, Shifter, and Bastable *Developing Mathematical Ideas casebooks: Reasoning Algebraically about Operations and Patterns, Functions and Change*.
- Extensive use of case studies help teachers develop their own mathematical ideas through thinking about how children encounter big ideas.
- Teachers must understand and represent ideas in multiple ways so that they can support students as they develop their understanding.

# By Design the Course Presses on Three Areas of Learning



# Strategies to Make Learning Explicit Within Each Domain

- Cases
- Problem solving through rich tasks
- Small group and whole group discussions
- Multiple representations
- Generalization
- Justification and proof

Content  
Knowledge  
for Teaching

- Interview students
- Use tasks from the cases with students
- Analyze students' work
- Examine teacher's moves in the cases
- Journaling
- Make visible the use of the Process Standard

Pedagogical  
Content  
Knowledge

- Small group and whole group discussions
- Participant presentation of problems
- Facilitator's modeling
- Facilitators "stepping in and stepping out"

Leadership  
Knowledge to  
Work With  
Adults

**Teaching Team => Mathematician + Mathematics Educator + Mathematics Specialist**

# Following the algebraic thread from arithmetic through formal algebra.

Relational Thinking  
to explore change in  
number sentences  
and move to  
generalization

Functional Thinking  
to explore patterns in  
co-variation and to  
represent  
generalization

Algebraic Thinking to  
use the knowledge  
and tools of algebra  
to explore, represent,  
communicate new  
ideas

# Developing Algebraic Reasoning

Rather than teaching algebra procedures, support learners to develop ways of thinking about arithmetic that are more consistent with the ways that students have to think to learn algebra successfully.

Adapted from the work of Thomas Carpenter

# Relational Thinking in an Arithmetic Context

Use one's knowledge of equality, operations, and relationships.

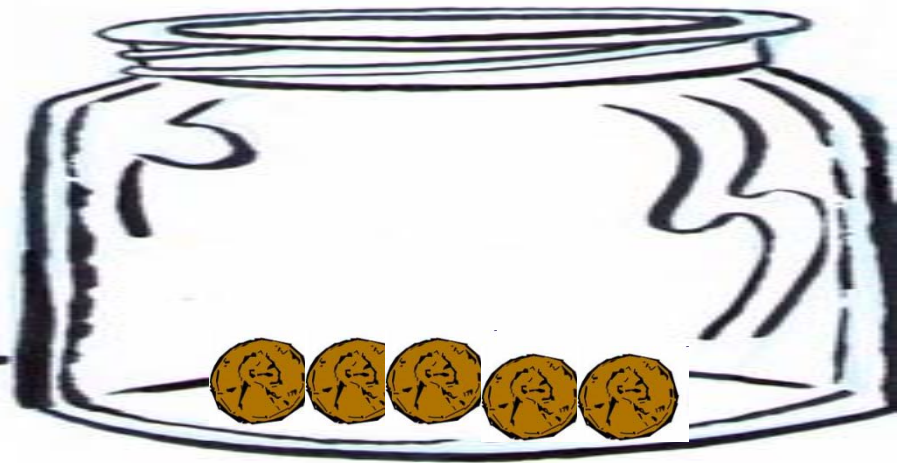
- $8 + 4 = \square + 5$
- T or F  
 $98 + 27 = 88 + 37$
- What value for  $m$  will make the sentence true?  
 $46 + 15 = 43 + m$

# Relational Thinking in an Algebraic Context

I was cleaning my closet and found an old jar that contained 5 pennies.

I decided to start saving some of the pennies I accumulate each day by putting 3 pennies in the jar daily.

## My Penny Jar

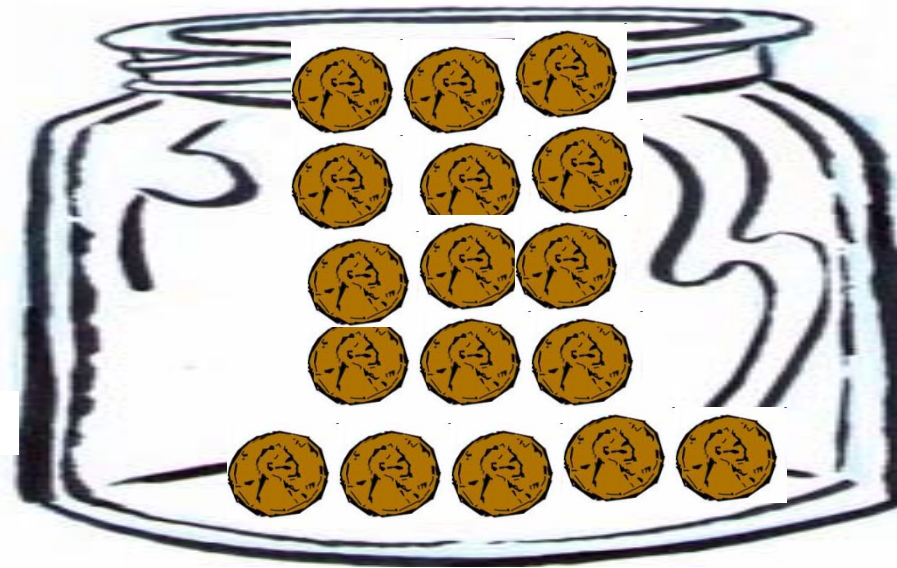




# Relational Thinking in an Algebraic Context

After 4 days how many pennies do I have?

## My Penny Jar



# Pennies for Hunger

Beth and Mario have penny jars in which to save their pennies for the school's "Pennies for Hunger" drive.

Mario's brother gave him 8 pennies to start and he saved 2 pennies each day. Beth started with an empty penny jar and she put 4 pennies in her jar each day.

Mario teased Beth that he would have more pennies in his jar to donate.

What do you think?



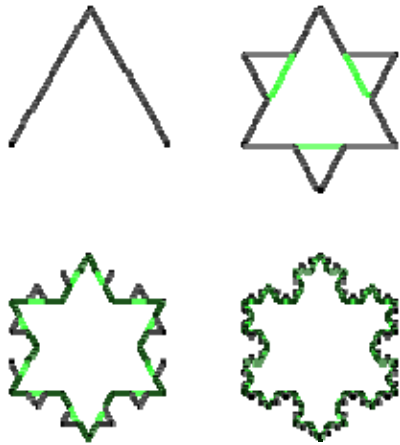
# Connecting to the Geometry Course

## Koch Snowflake Problem

- **Step One.**
  - Start with a large equilateral triangle and let each side have length one unit.
- **Step Two.**
  - Divide one side of the triangle into three equal parts and remove the middle section.
  - Replace it with two lines the same length as the section you removed.
  - Do this to all three sides of the triangle.

**Do it again and again.  
Do it infinitely many times and you have a fractal.**

**Koch Snowflake Problem**



- What is the perimeter of the new shape after 1 iteration, 3 iterations, n iterations?
- What is the area of the new shape after 1 iteration, 3 iterations, n iterations?

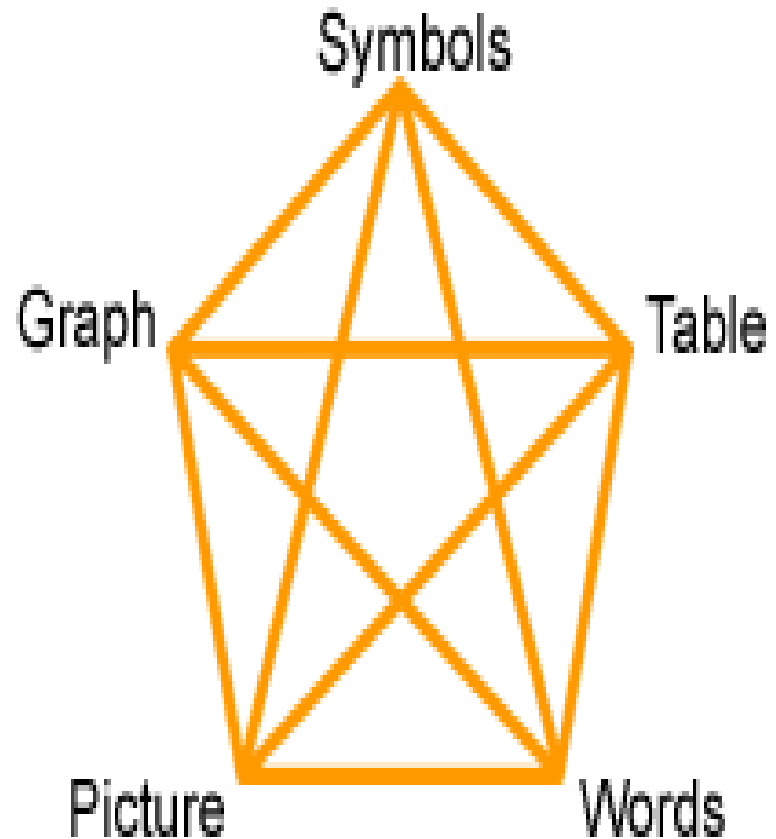
# NCTM PSSM

## VA Standards of Learning

Use and require students to use multiple representations.

Make explicit connections between how the concept is represented in tables, graphs, words, symbols, and pictures.

Because, different representations give us different information.



# Task from a Problem Set

- In Bugsville, USA at Swatum Elementary School, Mary a fifth grade student, decides to start a rumor that the town of Bugsville is going to declare September 14 as National Bug Day and is going to close schools for the day. She tells two students the rumor and also gives them instructions to each repeat the rumor to two students the next day who do not already know the rumor and that each of these new students is to repeat the rumor to two more students on the third day, and so on.
- How many new students will know the rumor on day 10? If Mary starts her rumor on September 1 and there are 8000 students in the school district will all the students hear the rumor before September 14, support your response? Is there a generalization that Mary can pass along to her friends in other school divisions so they can figure out how many days it will take to spread a rumor?
- What patterns do you notice in the data, how is this pattern reflected in the graphical representation of the data?

# Push to Non-Linear Relationships

Day	Number of New People Informed at Beginning of Each Day	Total Number of People Who Now Know
1	0	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63
7	64	127
8	128	255
9	256	511
10	512	1023
n	$2^{n-1}$	$2^n - 1$

- Task has multiple entry points
- Data can be represented in multiple ways
- Participants can represent the nth term verbally or in symbols

# Related Task

It is really, really hot but Beth needs to bale the hay on her farm. She decides to pace herself so she has decided to bale  $\frac{1}{2}$  of the hayfield when she is most rested and take a break, then she will bale  $\frac{1}{4}$  of the hay field and take another break, then bale  $\frac{1}{8}$  of the hay field and take another break. She will continue this until all the hay is baled.

When will the entire hay field be complete? Show the justification for your response? Is there a generalization that can be used to determine what part of the entire field has been baled at any break?



# Related Task

After break n	How much of the field is baled during the baling period n	Total amount of field baled
1	1/2	1/2
2	1/4	3/4
3	1/8	7/8
4	1/16	15/16
5	1/32	31/32
6	1/64	63/64
n	$1/2^n$	$(2^n - 1) / 2^n$

- Task has multiple entry points
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Email from Algebra Teacher turned middle school coach enrolled in the Functions Algebra course who was asked to take over an algebra class for a teacher who had to leave school quickly one day.

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I had been in this class before with the penny jar problem so they were not unfamiliar with my teaching style. I had to come up with something very quickly and I have sworn to myself I will not teach "out of the book" anymore so I quickly made up a word problem about renting a surfboard. I gave them four different graphs and told the groups to tell me everything they could about each graph and then tell me the best deal. They came up with steepness of the line representing the cost per hour so the steeper the line the greater the cost. An initial fee for the surfboard was in a couple of the graphs, a constant charge of \$6 was in one of the graphs. They came up with all of the points I was hoping to cover.

After all this discussion, I gave them 2 points (which we must do for our SOLs) and asked if they could tell me the slope between these two points. I told them they had to show it 2 different ways but I was not going to tell them anything. After all of our discussion about the surfboard, a few of the groups (one with the smartest boy in the class) inverted the slope - instead of  $9/2$ , they put  $2/9$ . They had used graphing to show their work. While showing their work, one of them made the statement the smaller the fraction the steeper the line. The group sat down and I wrote that statement on the board. Now many of the other groups were disturbed by this but this boy is seldom wrong. I told them to use the work we had done with the word problem to prove or disprove their view. There was so much debate in this room.

Eventually the correct view point was proven and much was learned by all, including me. This was not a well thought out lesson but the students were engaged either describing all they could figure out or proving their point of view to others. I HAD THE BEST TIME!!!!!! ....I will just keep on trying to put meaning into my work.

# Functions and Algebra Quote

“ I found that this Math Specialist Program placing (sp.) a lot of emphasis on understanding why a rule is the rule. In the beginning of this class I related the song “I will survive” to my high school algebra experience. During the beginning of this class, there was no doubt that this was going to be my “theme song” this time around as well. With all the emotional stress, mathematical challenges, and mathematical talk I’ve endured during this class I must say my theme song has retired and a new one is in town---”I believe I can prove, I believe I can justify.” {goes to the tune “I believe I can fly” by R. Kelly]

Spring 2005

# Functions and Algebra Quote

“This two week intensive algebra and functions mathematics course was well worth the time and effort. I have gained a firmer understanding of algebraic functions, patterns, and properties. I feel more confident and competent with working with all grade levels, K-8, on mathematics.”

Summer Institute 2006

# Functions and Algebra Quote

“During the reading I became totally fascinated with tables and their relationships with graphs and slopes. In Chapter 3 I really enjoyed Drew. The funny thing is as much as I HATE the focus questions, if not for having to answer I would not have understood Drew’s thinking.”

Summer Institute 2006

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