Summary of Research on Experiences Intended to Deepen Teachers' Mathematics Content Knowledge

Studies of two types of experiences were included in the review of research related to deepening teachers' mathematics content knowledge. First, 22 studies investigated the effects of teachers' experience in professional development programs that had deepening teachers' mathematics content knowledge as a goal. Second, 7 studies examined teaching practice as a context for teachers to deepen their mathematics content knowledge. One study fit into both categories, yielding a total of 28 studies in the review.

Information on how these studies were identified and a summary of the review methodology can be found at:

http://www.mspkmd.net/index.php?page=02_4a-3c2

Effects of Programs Aimed at Deepening Teachers' Mathematics Content Knowledge
Studies of the effects of 22 different interventions designed to deepen teachers' mathematics content knowledge were reviewed. Information about the research studies is displayed in Table 1. Information about the interventions examined in the 22 studies is shown in Table 2.

In all 22 studies, participating teachers' mathematics content knowledge increased. At a minimum, these results provide existence proofs that experiences aimed at deepening teachers' mathematics content knowledge can achieve that goal. It is important, however, to bear in mind that studies with positive effects are probably more likely to be submitted, and possibly more likely to be accepted, for publication than those with no effects or negative effects.

The diversity of the programs investigated across these 22 studies suggests that there are a variety of effective ways of structuring and delivering experiences to deepen teachers' mathematics content knowledge. The programs also differed in the grade range of participating teachers and the mathematics content strands that were addressed. Positive effects were found for experiences with teachers from elementary, middle, and high school grades variously targeting algebra; data analysis, probability and statistics; geometry; measurement; number and operations; and problem solving and representation. On the whole, there is more empirical evidence regarding interventions for middle grades[1] and elementary[2] teachers than for high school teachers.[3]

In some cases, the interventions were described in detail,[4] which is helpful for understanding teachers' experiences and interpreting the link between the intervention and the effects on teachers' mathematics content knowledge. In several cases, however, the intervention was described only partially,[5] making it more difficult to support these interpretations.

All of the programs in the reviewed studies consisted of either a long-term course or an intensive workshop lasting at least two days. Four of the programs included semester-long courses,[6] 10 included intensive summer workshops lasting at least a week,[7] and 7 included workshops held during the school year.[8] These courses or workshops all focused on a specific topic in mathematics and situated teachers' conceptual learning within the work they do in classrooms. Many of the programs that were studied included follow-up sessions or seminars in addition to the courses or workshops.[9] Several of the programs included classroom observations and/or follow-up interviews designed to support teachers' application of what they were learning to their practice.[10] For example, one program involved long-term, on-site professional development

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[1] Basile et al., 2006; Basista & Mathews, 2002; Clark & Schorr, 2000; Cochran, Mayer, & Mullins, 2007; Consortium for Policy Research in Education (CPRE), 2007; Dole, Clark, Wright, Hilton, & Roche, 2008; Garner-Gilchrist, 1993; Geer, 2001; Goldsmith & Seago, 2007; Santagata, 2009; Sowder, Phillip, Armstrong, & Schappelle, 1998; Swafford, Jones, & Thornton, 1997; Swafford, Jones, Thornton, Stump, & Miller, 1999; Vale & McAndrew, 2008; Weaver & Dick, 2009.

[2] Basista & Mathews, 2002; Benken & Brown, 2008; Cochran et al., 2007; Dole et al., 2008; Ellington, Whitenack, Inge, Murray, & Schneider, 2009; Franke, Carpenter, Fennema, Ansell, & Behrend, 1998; Garner-Gilchrist, 1993; Geer, 2001; Hill & Ball, 2004; Hughes & Gilbert, 2007; Stecher & Mitchell, 1995; Swafford et al., 1997; Swafford et al., 1999; Weaver & Dick, 2009.

[3] Basista & Mathews, 2002; Cochran et al., 2007; Dole et al., 2008; Geer, 2001; Goldsmith & Seago, 2007, McCoy, Hill, Sack, Papakonstantinou, & Parr, 2007; Vale & McAndrew, 2008; Weaver & Dick, 2009.

[4] CPRE, 2007; Ellington et al., 2009; Franke et al., 1998; Hughes & Gilbert, 2007; Santagata, 2009; Sowder et al., 1998; Stecher & Mitchell, 1995; Strom, 2006; Vale & McAndrew, 2008; Weaver & Dick, 2009.

[5] Basista & Mathews, 2002; Benken & Brown, 2008; Clark & Schorr, 2000; Cochran et al., 2007; Dole et al., 2008; Garner-Gilchrist, 1993; Geer, 2001; Goldsmith & Seago, 2007; Hill & Ball, 2004; McCoy et al., 2007; Swafford et al., 1997; Swafford et al., 1999.

[6] Clark & Schorr, 2000; Garner-Gilchrist, 1993; Hughes & Gilbert, 2007; Strom, 2006.

[7] Basile et al., 2006; Basista & Mathews, 2002; Cochran et al., 2007; Ellington et al., 2009; Geer, 2001; Hill & Ball, 2004; McCoy et al., 2007; Stecher & Mitchell, 1995; Swafford et al., 1997; Swafford et al., 1999; Weaver & Dick, 2009.

[8] Dole et al., 2008; Franke et al., 1998; Goldsmith & Seago, 2007; Santagata, 2009; Sowder et al., 1998; Stecher & Mitchell, 1995; Vale & McAndrew, 2008.

[9] Basile et al., 2006; Basista & Mathews, 2002; Franke et al., 1998; Geer, 2001; Hill & Ball, 2004; Santagata, 2009; Swafford et al., 1997; Swafford et al., 1999.

[10] Basista & Mathews, 2002; Franke et al., 1998; Hughes, 2007; Sowder et al., 1998, Strom, 2006.

occurring bimonthly across the school year;[11] another included intensive coaching and a deep discussion of assessment items.[12]

In addition to the fact that nearly all of the 22 studied programs were fairly extensive, requiring at least one week of commitment (typically more) and multiple meetings, in nearly all cases the teachers were indicated to be volunteers.[13] In at least two cases, the teachers were also screened prior to selection for participation in the interventions to ensure that they were committed to changing their teaching practice.[14] Generalizability of the findings from these studies must be considered in light of these parameters, because the populations that these teachers represent are limited to teachers willing and able to commit to participation in such extensive interventions.

Nearly all of the programs that were studied included attention to both disciplinary content knowledge and pedagogical content knowledge, although in varying degrees of emphasis.[15] The remaining programs addressed in one case both teachers' knowledge of ways of knowing in mathematics and their pedagogical content knowledge[16] and in the other case only disciplinary content knowledge.[17] Across the studies the level of disciplinary content knowledge addressed varied, including student-level content ideas, more advanced disciplinary content, and a more profound understanding of fundamental mathematics ideas. In addition, the programs attended to different aspects of pedagogical content knowledge and ways of knowing in mathematics. It is not possible from this small set of studies, with varying goals for deepening teachers' content knowledge, to know what kinds of programs are the most efficient or effective for achieving particular goals.

The Evidentiary Base for Claims about Programs Aimed at Deepening Teachers' Mathematics Content Knowledge

It is important to recognize that particular features of the programs, although described in detail in some cases and logically tied to the reported impacts on teachers' mathematics content knowledge, were not investigated in any of the studies, except one described below, through

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[11] Benken & Brown, 2008.

[12] CPRE, 2007.

[13] Benken & Brown, 2008; Clark & Schorr, 2000; Cochran et al., 2007; CPRE, 2007; Ellington et al., 2009; Franke et al., 1998; Garner-Gilchrist, 1993; Geer, 2001; Goldsmith & Seago, 2007; Hughes & Gilbert, 2007; McCoy et al., 2007; Sowder et al., 1998; Strom, 2006; Swafford et al., 1997; Swafford et al., 1999; Vale & McAndrew, 2008; Weaver & Dick, 2009.

[14] Garner-Gilchrist, 1993; Sowder et al., 1998.

[15] Basile et al., 2006; Basista & Mathews, 2002; Benken & Brown, 2008; Clark & Schorr, 2000; Cochran et al., 2007; CPRE, 2007; Dole et al., 2008; Ellington et al., 2009; Franke et al., 1998; Garner-Gilchrist, 1993; Geer, 2001; Goldsmith & Seago, 2007; Hill & Ball, 2004; Hughes & Gilbert, 2007; McCoy et al., 2007; Santagata, 2009; Sowder et al., 1998; Vale & McAndrew, 2008; Weaver & Dick, 2009.

[16] Stecher & Mitchell, 1995.

[17] Strom, 2006.

either systematic or naturalistic variation. Findings in these studies can only be understood to result from teachers' experience of the programs as a whole.

Different measures of teachers' content knowledge were used across the studies, and some intended impacts on teachers' mathematics content knowledge were not measured. As a result, it is not possible to identify whether features of one program may be more or less effective for a particular purpose than features of another program. Claims that some features are important for deepening teachers' mathematics content knowledge are suggested to some extent by their presence in the multiple programs studied. The importance of these features in deepening particular facets of teachers' content knowledge was supported on logical or theoretical grounds, but not empirically. One study[18] did examine variations in teachers' experiences of different professional development workshops in relation to their content knowledge gains. These analyses suggested that summer institutes of greater duration, and those that focused on mathematical analysis, reasoning, and communication had larger impacts on teachers' mathematics content knowledge. The researchers advised caution with respect to these results due to the fact that only about one-fourth of the eligible institutes agreed to participation in the study, so bias in the samples of professional development experiences and teachers could have affected the findings.

Another important consideration for interpreting the results of several of the studies was delivery of the interventions by the researchers,[19] who in some cases were also the developers of the interventions. When researchers develop and deliver interventions, it is more likely that they are implemented as intended. However, these researchers, whether developers or deliverers, may have a vested interest in study outcomes, potentially introducing biases toward evidence of intended outcomes. Also, implementation of the programs may have included aspects that remained implicit and would therefore not appear in researchers' descriptions, making replication of the interventions very difficult.

All of these studies but one used either a pre-post design to measure changes in teachers' content knowledge or traced changes in teachers' content knowledge over multiple points in time. In the one study that used a post-only design[20], the teachers reported that the intervention had influenced the aspects of their content knowledge that were measured. However, only one of these studies used comparison groups of teachers who did not participate in the professional development programs.[21] Given the experience levels of many of the participating teachers, the extent of professional development provided, and the nature of the measured changes, it is certainly reasonable to argue that the changes resulted from the interventions. However, without comparisons to other teachers these claims are not solidly grounded in empirical evidence. For

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[18] Hill & Ball, 2004.

[19] Basista & Mathews, 2002; Benken & Brown, 2008; Clark & Schorr, 2000; Cochran et al., 2007; Franke et al., 1998; Goldsmith & Seago, 2007; McCoy et al., 2007; Santagata, 2009; Sowder et al., 1998; Strom, 2006; Swafford et al., 1997; Swafford et al., 1999; Vale & McAndrew, 2008.

[20] Stecher & Mitchell, 1995.

[21] Goldsmith & Seago, 2007.

example, it is possible that the teachers might perform better on a measure of content knowledge on a post-test simply because they had completed it previously, in two cases[22] only a few weeks earlier. The use of multiple measures addresses this concern to some extent, as in Swafford and colleagues' study[23] in which the participating teachers performed better in three different content areas, and on three separate measures of knowledge of geometry, following treatment. Similarly, six other studies used both written instruments and interviews with teachers to measure teacher content knowledge, and in one additional case a combination of interviews and observations.[24]

[22] Basista & Mathews, 2002; Cochran et al., 2007.

[23] Swafford et al., 1997; Swafford et al., 1999.

[24] Benken & Brown, 2008; Clark & Schorr, 2000; Franke et al., 1998; Hughes & Gilbert, 2007; Sowder et al., 1998; Stecher & Mitchell, 1995; Strom, 2006.

Table 1
Studies of Interventions to Deepen Teachers' Mathematics Content Knowledge: Study Characteristics

Name of Study	Purpose of Study		Data types		Knowledge Outcomes			Measures of Teacher Content Knowledge				Measurement Description		
	Program Evaluation	Providing Examples	Quantitative	Qualitative	Disciplinary Content	Ways of Knowing	Pedagogical Content	Assessments	Interviews	Observations	Other Approach	Validity	Reliability	Triangulation
The veritable quandary of measuring teacher content knowledge in a math and science partnership. (Basile et al., 2005)	•			•	•		•	•						
Integrated science and mathematics professional development programs (Basista & Mathews, 2002)	•		•	•	•		•	•		•				
Moving beyond the barriers: A re-defined, multi-leveled partnership approach to mathematics teacher education (Benken & Brown, 2008)	•		•	•	•		•		•		•			•
Teachers' evolving models of the underlying concepts of rational number (Clark & Schorr, 2000)	•			•	•		•	•	•	•				
The impact of inquiry-based mathematics courses on content knowledge and classroom practice. (Cochran et al., 2007)	•		•		•		•	•						
The El Paso staff developer study: Overview and initial findings of the Math/Science Partnership (MSP) Middle Grades Initiative, 2005-2007 (CPRE, 2007)	•		•		•		•	•						
Eliciting growth in teachers' proportional reasoning: Measuring the impact of a professional development program (Dole et al., 2008)		•		•	•		•				•			
Assessing K-5 teacher leaders' mathematical understanding: What have the test makers and the test takers learned? (Ellington et al., 2009).	•		•	•	•		•	•						
Understanding teachers' self-sustaining, generative change in the context of professional development (Franke et al., 1998)		•		•	•		•		•	•		•	•	•
Mathematics institute: An inservice program for training elementary school teachers (Garner-Gilchrist, 1993)		•	•		•		•	•						
Science and mathematics professional development at a liberal arts university: Effects on content knowledge, teacher confidence and strategies, and student achievement (Geer, 2001)	•		•		•		•	•				•		

Table 1 (continued)
Studies of Interventions to Deepen Teachers' Mathematics Content Knowledge: Study Characteristics

	Purpose of Study		Data types		Knowledge Outcomes			Measures of Teacher Content Knowledge				Measurement Description		
Name of Study	Program Evaluation	Providing Examples	Quantitative	Qualitative	Disciplinary Content	Ways of Knowing	Pedagogical Content	Assessments	Interviews	Observations	Other Approach	Validity	Reliability	Triangulation
Tracking teachers' learning in professional development centered on classroom artifacts (Goldsmith & Seago, 2007)	•		•	•	•		•	•			•			
Learning mathematics for teaching: Results from California's mathematics professional development Institutes (Hill & Ball, 2004)	•		•		•		•	•				•	•	
The P-5 mathematics endorsement; Impacts and lessons learned (Hughes & Gilbert, 2007)	•		•	•	•		•	•	•	•				•
Strengthening mathematics teachers' pedagogical content knowledge through collaborative investigations in combinatorics (McCoy et al., 2007)	•		•	•	•		•	•						•
Designing video-based professional development for mathematics teachers in low-performing schools (Santagata, 2009)		•		•	•		•	•			•			
Middle-grade teachers' mathematical knowledge and its relationship to instruction (Sowder et al., 1998)		•		•	•		•	•	•	•		•	•	•
Vermont teachers' understanding of mathematical problem solving and "good" math problems (Stecher & Mitchell, 1995)	•			•		•	•	•	•			•	•	•
The role of covariational reasoning in learning and understanding exponential functions (Strom, 2006)	•		•	•	•			•	•			•	•	•
Increased knowledge in geometry and instructional practice (Swafford et al., 1997) The impact on instructional practice of a teacher change model (Swafford et al., 1999)	•		•		•		•	•[a]				•	•	•

Table 1 (continued)
Studies of Interventions to Deepen Teachers' Mathematics Content Knowledge: Study Characteristics

	Purpose of Study		Data types		Knowledge Outcomes			Measures of Teacher Content Knowledge				Measurement Description		
Name of Study	Program Evaluation	Providing Examples	Quantitative	Qualitative	Disciplinary Content	Ways of Knowing	Pedagogical Content	Assessments	Interviews	Observations	Other Approach	Validity	Reliability	Triangulation
Deepening the mathematical knowledge of secondary mathematics teachers who lack tertiary mathematics qualifications (Vale & McAndrew, 2008)	•			•	•		•	•			•			
Oregon Mathematics Leadership Institute Project: Evaluation results on teacher content knowledge, implementation fidelity, and student achievement (Weaver & Dick, 2009)	•		•		•		•	•						

[a] Indicates use of an existing measure that was not developed specifically for the purpose of this study.

Table 2
Studies of Interventions to Deepen Teachers' Mathematics Content Knowledge: Intervention Characteristics

Name of Study	Grade Level	Intervention[a]				Content/Process Strand							
		Full Description	Teacher Involvement Voluntary	STEM Faculty Involved	Researcher(s) Involved	Number and Operations	Algebra	Geometry	Measurement	Data, Probability, Statistics	Communication	Problem Solving	Representation
The veritable quandary of measuring teacher content knowledge in a math and science partnership. (Basile et al., 2005)	6–8	N	?	Y	N	•	•	•	•	•	•	•	•
Integrated science and mathematics professional development programs (Basista & Mathews, 2002)	4–10	N	?	Y	Y	•	•	•		•	•	•	•
Moving beyond the barriers: A re-defined, multi-leveled partnership approach to mathematics teacher education (Benken & Brown, 2008)	K–5	N	?	Y	Y	•	•	•	•	•	•	•	•
Teachers' evolving models of the underlying concepts of rational number (Clark & Schorr, 2000)	6–8	N	Y	?	Y	•							
The impact of inquiry-based mathematics courses on content knowledge and classroom practice. (Cochran et al., 2007)	K–12	N	?	Y	Y		•						
The El Paso staff developer study: Overview and initial findings of the Math/ Science Partnership Middle Grades Initiative, 2005-2007 (CPRE, 2007)	6–8	Y	Y	Y	N	•	•	•					
Eliciting growth in teachers' proportional reasoning: Measuring the impact of a professional development program (Dole et al., 2008)	6–8	N	?	?	N	•							
Assessing K-5 teacher leaders' mathematical understanding: What have the test makers and the test takers learned? (Ellington et al., 2009)	K–5	Y	?	Y	?	•							
Understanding teachers' self-sustaining, generative change in the context of professional development (Franke et al., 1998)	1–3	Y[b]	Y	?	Y	•		•					
Mathematics institute: An inservice program for training elementary school teachers (Garner-Gilchrist, 1993)	4–8	N	Y	Y	N		•	•		•			
Science and mathematics professional development at a liberal arts university: Effects on content knowledge, teacher confidence and strategies, and student achievement (Geer, 2001)	4–9	N	Y	Y	N	•	•	•	•	•			

Table 2 (continued)
Studies of Interventions to Deepen Teachers' Mathematics Content Knowledge: Intervention Characteristics

Name of Study	Grade Level	Intervention[a] Full Description	Intervention[a] Teacher Involvement Voluntary	Intervention[a] STEM Faculty Involved	Intervention[a] Researcher(s) Involved	Content/Process Strand: Number and Operations	Algebra	Geometry	Measurement	Data, Probability, Statistics	Communication	Problem Solving	Representation
Tracking teachers' learning in professional development centered on classroom artifacts (Goldsmith & Seago, 2007)	6–12	N	?	Y	Y		•						
Learning mathematics for teaching: Results from California's mathematics professional development institutes (Hill & Ball, 2004)	K–6	Y[b]	?	Y	N	•					•		•
The P-5 mathematics endorsement; Impacts and lessons learned (Hughes & Gilbert, 2007)	K–5	Y	Y	Y	N	•	•	•		•	•	•	•
Strengthening mathematics teachers' pedagogical content knowledge through collaborative investigations in combinatorics (McCoy et al., 2007)	9–12	N	Y	Y	Y							•	
Designing video-based professional development for mathematics teachers in low-performing schools (Santagata, 2009)	6	Y	N	Y	Y	•	•						
Middle-grade teachers' mathematical knowledge and its relationship to instruction (Sowder et al., 1998)	6–8	Y	Y	Y	Y	•							
Vermont teachers' understanding of mathematical problem solving and "good" math problems (Stecher & Mitchell, 1995)	4	N	N	?	N							•	
The role of covariational reasoning in learning and understanding exponential functions (Strom, 2006)	6-12	Y	Y	Y	Y		•						
Increased knowledge in geometry and instructional practice (Swafford et al., 1997) The impact on instructional practice of a teacher change model (Swafford et al., 1999)	4–8	N	Y	Y	Y		•	•		•			

Table 2 (continued)
Studies of Interventions to Deepen Teachers' Mathematics Content Knowledge: Intervention Characteristics

Name of Study	Grade Level	Intervention[a] Full Description	Teacher Involvement Voluntary	STEM Faculty Involved	Researcher(s) Involved	Content/Process Strand Number and Operations	Algebra	Geometry	Measurement	Data, Probability, Statistics	Communication	Problem Solving	Representation
Deepening the mathematical knowledge of secondary mathematics teachers who lack tertiary mathematics qualifications (Vale & McAndrew, 2008)	6–12	Y	Y	Y	Y		•			•			
Oregon Mathematics Leadership Institute Project: Evaluation results on teacher content knowledge, implementation fidelity, and student achievement (Weaver & Dick, 2009)	K–12	Y	?	Y	N	•	•	•	•	•	•	•	•

[a] Y = Yes, N = No, ? = Not clear from document
[b] Includes reference with full description.

Teaching Practice as a Context for Deepening Teachers' Mathematics Content Knowledge

Also included in this set were seven studies that investigated whether teachers can deepen their mathematics content knowledge as a result of their teaching practice itself. In all seven of these studies, the teachers had been, or were simultaneously, involved in an experience to support their mathematics teaching practice. Although each of these studies included an intervention, the intervention may not have been directly focused on deepening teachers' mathematics content knowledge. In only one case, the research study by Franke and colleagues[25] (also included among the studies described in the previous section), was this intervention known to focus on deepening teachers' content knowledge. Table 3 provides information about the research studies, and Table 4 displays information about the interventions examined in these seven studies.

All seven of the studies that examined teaching practice as a contributor to deepening teachers' content knowledge documented positive effects. The seven studies each investigated a different aspect of teaching practice, suggesting that multiple aspects of practice may serve as potential contributors to content knowledge gains. Four of the studies examined elementary school teachers, together spanning grades 1–5, and all of these focused on number and operations.[26] The other three examined secondary school teachers, all focusing on algebra.[27] Although the number of studies is small, there is at least a suggestion that teacher learning of content from practice is possible at multiple grade levels. It is worth noting that all of the studies focused on teachers' learning about a very familiar strand of mathematics for the grade levels being examined. However, no empirical evidence is available to suggest any differences regarding teachers learning one strand of mathematics content versus others, or about teachers learning of more or less familiar content, from their practice.

The seven studies employed a variety of methods to examine aspects of participating teachers' classroom teaching practice, which is the independent variable of interest in these studies. All used individual interviews to examine the teaching practices of participating teachers, and three of the studies also examined teaching practice through formal of informal group interviews occurring during meetings with groups of teachers to discuss their teaching practices.[28] Classroom observations were conducted in four of the studies.[29] Samples of student work from the teachers' classrooms were used in two of the studies as a way to focus on student strategies and thinking within teachers' classroom practice.[30]

[25] Franke et al., 1998.

[26] Empson, 1999; Featherstone, Smith, Beasley, Corbin, & Shank, 1995; Franke et al., 1998; Lin, 2002.

[27] Chazan, Yerushalmy, & Leikin, 2008; Miller, 1991.

[28] Empson, 1999; Franke et al., 1998; Miller, 1991.

[29] Featherstone et al., 1995; Franke et al., 1998; Lin, 2002; Noh & Kang, 2007.

[30] Lin, 2002; Miller, 1991.

In five of these studies that investigated teacher learning from practice, the main outcome of interest was pedagogical content knowledge;[31] four studies examined teacher learning of disciplinary content as an outcome,[32] and one of these programs also addressed knowledge of ways of knowing in mathematics, although impacts on this type of knowledge were not measured.[33] In all seven studies, at least some positive results were reported for each outcome that was investigated, suggesting that teacher learning from practice may include multiple facets of mathematics content knowledge. However, it is worth noting that studies with positive effects are probably more likely to be submitted, and may be more likely to be accepted, for publication than those with no effects or negative effects.

The Evidentiary Base for Claims about Teaching Practice as a Context for Deepening Teachers' Mathematics Content Knowledge

The main purposes of the seven studies of teaching practice were to illustrate and substantiate how teachers can learn mathematics content knowledge through their teaching practice. Each of the seven studies involved only a small number of teachers, collected only post-experience data, and did not investigate specific variations in teaching practice, so claims regarding causation or generalizability can be only weakly supported. The common finding in these studies that teaching practice presents a context in which teachers can learn mathematics content suggests, however, that efforts to deepen teachers' content knowledge might expand their impact by attending to the context of teaching practice as a site for learning. By providing appropriate structures, resources, and opportunities to support learning, professional development efforts intended to deepen teachers' mathematics content knowledge might take advantage of teachers' ongoing work in their schools and classrooms to bolster their content learning.

Each of the seven studies provided examples from teacher interviews, and four also used examples from classroom observations, to illustrate teachers' learning from their practice. Because this is a fairly new area of investigation, the illustrations of teachers' mathematics content learning in these exploratory studies are a key contribution to building theory about teacher learning from practice. Three of the studies did not present an analysis of data over time that would clearly support claims of teacher learning, although they did link the post-experience outcome data to the teachers' experiences with particular teaching practices.[34]

A few issues regarding validity and generalizability in these studies should also be noted. In three of the studies, systematic methods of analyses were described that included important elements such as establishing reliability among coders and member checking through post-observation interviews. Their overall study designs were aligned with the exploratory and illustrative nature of the research.[35] Methods for selecting the examples that were presented, or

[31] Empson, 1999; Featherstone et al., 1995; Franke et al., 1998; Lin, 2002; Miller, 1991.

[32] Chazan et al., 2008; Empson, 1999; Featherstone et al., 1995, Noh & Kang, 2007.

[33] Featherstone et al., 1995.

[34] Chazan et al., 2008; Empson, 1999; Miller, 1991.

[35] Featherstone et al., 1995; Franke et al., 1998; Lin, 2002.

for seeking data that are discrepant with the findings, were not apparent in the other four studies, leaving questions about the completeness of interpretation of the full range of data in these studies.[36] In at least 3 of the 7 studies, researcher biases toward particular findings, arising because the researchers conducted interventions with the teachers, may have been present.[37]

As exploratory studies, generalizability was not a primary concern. It is important to bear in mind that the teachers participating in these studies were committed to programs to support improvement and/or investigation of their practice, and that much of their learning may have derived not only from changes in their practice but also from the opportunities they had to reflect on their practice with colleagues and mathematics educators.

Findings in these seven studies, commensurate with the purpose of exploratory research, provide a basis for theorizing about teacher learning from practice, and are intriguing as hypotheses to investigate further. Causality is not strongly established by the empirical evidence. Generalizability is supported by thorough descriptions that can be compared to the readers' own experiences with teachers, but not through systematic or representative sampling from a defined population.

[36] Chazan et al., 2008; Empson, 1999; Miller, 1991, Noh & Kang, 2007.

[37] Franke et al., 1998; Lin, 2002; Miller, 1991.

Table 3
Studies of Deepening Teachers' Mathematics Content Knowledge Through Their Instructional Practice: Study Characteristics

	Purpose of Study		Data Types		Knowledge Outcomes			Measures of Teacher Content Knowledge				Measurement Description		
Name of Study	Program Evaluation	Providing Examples	Quantitative	Qualitative	Disciplinary Content	Ways of Knowing	Pedagogical Content	Assessments	Interviews	Observations	Other Approach	Validity	Reliability	Triangulation
An analytic conception of equation and teachers' views of school algebra (Chazan et al., 2008)		•		•	•				•					
Considerations of systemic change and teachers' knowledge of students' novel strategies for whole-number operations (Empson, 1999)		•		•	•		•		•					
Expanding the equation: Learning mathematics through teaching in new ways (Featherstone et al., 1995)		•		•	•	•	•		•	•		•	•	•
Understanding teachers' self-sustaining, generative change in the context of professional development (Franke et al., 1998)		•		•	•		•		•	•		•	•	•
On enhancing teachers' knowledge by constructing cases in classrooms (Lin, 2002)		•		•			•		•	•	•	•	•	•
Constructing pedagogical content knowledge from students' writing in secondary school (Miller, 1991)		•		•			•		•		•			
Exploring the idea of curriculum materials supporting teacher knowledge (Noh & Kang, 2007)		•		•	•				•	•				

Table 4
Studies of Deepening Teachers' Mathematics Content Knowledge Through Their Instructional Practice: Intervention Characteristics

Name of Study	Grade Level	Intervention[a]				Content Strand	
		Full description	Teacher involvement voluntary	STEM faculty involved	Researcher(s) involved	Number and operations	Algebra
An analytic conception of equation and teachers' views of school algebra (Chazan et al., 2008)	6–12	Y	?	?	Y		●
Considerations of systemic change and teachers' knowledge of students' novel strategies for whole-number operations (Empson, 1999)	3–5	N	N	N	N	●	
Expanding the equation: Learning mathematics through teaching in new ways (Featherstone et al., 1995)	2–3	N	Y	Y	N	●	
Understanding teachers' self-sustaining, generative change in the context of professional development (Franke et al., 1998)	1–3	Y[b]	Y	?	Y	●	
On enhancing teachers' knowledge by constructing cases in classrooms (Lin, 2002)	1	Y	?	Y	Y	●	
Constructing pedagogical content knowledge from students' writing in secondary school (Miller, 1991)	9–12	N	?	?	Y		●
Exploring the idea of curriculum materials supporting teacher knowledge (Noh & Kang, 2007)	9–12	N	Y	?	Y		●

[a] Y = Yes, N = No, ? = Not clear from document
[b] Includes reference with full description.

Bibliography for Summary of Research on Experiences Intended to Deepen Teachers' Mathematics Content Knowledge

Additional information on how these studies were identified and reviewed, a summary of the methodology can be found at:

http://www.mspkmd.net/index.php?page=02_4a-3c2

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